

1. What does it mean to “solve” a quadratic equation? Explain in a few sentences.

find which x-values make the equation true (x-intercepts)

2. What is the difference between a quadratic functions and a linear function? Explain in a few sentences.

quadratic has x^2 (degree 2) linear is just x (degree 1)

3. Decide whether each of the following methods *always*, *sometimes*, or *never* work for solving quadratic equations.

a) Square Root Method *sometimes*
when no x-value

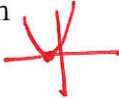
b) Factoring *sometimes*
when you can factor

c) Completing the Square

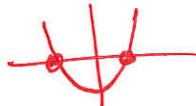
d) Quadratic Formula *Always*

4. Determine whether each of the following are possibilities for the solution set of a quadratic equation. (Yes/No). If yes, draw a picture to illustrate each solution.

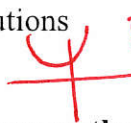
a) Only one solution

Yes  touches once

b) Two real solutions

Yes  touches twice

c) No real number solutions

Yes  doesn't touch x-axis

d) Three real solutions

No

Solve each equation using any method you would like.

5. $9k^2 + 3k - 6 = 0$

$(3k^2 + k - 2) = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $\frac{-1 \pm \sqrt{1 - 4(3)(-2)}}{2(3)}$

$(3k^2 + 3k)(-2k - 2) = 0$

$(3k(k+1) - 2(k+1))$

$(k+1)(3k-2) = 0$

$k+1=0 \Rightarrow k=-1$
 $3k-2=0 \Rightarrow k=\frac{2}{3}$

$k = -1$
 $k = \frac{2}{3}$

7. $12x^2 - 8 = 16$

$+8 +8$

$\frac{12x^2}{12} = \frac{24}{12}$

$\sqrt{x^2} = \sqrt{2}$

$x = \pm \sqrt{2}$

9. $2x^2 - 10x = 0$

$2x(x-5) = 0$

$\frac{2x}{2} = \frac{0}{2} \Rightarrow x=0$
 $\frac{x-5}{+5} = \frac{0}{+5} \Rightarrow x=5$

$x = 0 \quad x = 5$

6. $w^2 - 9w = -14$

$w^2 - 9w + 14 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $\frac{9 \pm \sqrt{81 - 4(1)(14)}}{2(1)}$

$(w-7)(w-2) = 0$

$w-7=0 \quad w-2=0$

$w = 7, w = 2$

8. $3x^2 - 6x = 15$

$\frac{3x^2 - 6x - 15}{3} = 0 \Rightarrow x^2 - 2x - 5 = 0$

$a=1 \quad b=-2 \quad c=-5$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $\frac{2 \pm \sqrt{4 - 4(1)(-5)}}{2(1)}$

$2 \pm \sqrt{(-2)^2 - 4(1)(-5)}$

$\frac{2 \pm \sqrt{24}}{2} = \frac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6} = x$

$\sqrt{24} = 2\sqrt{6}$

10. $5(x-6)^2 = 50$

$\sqrt{(x-6)^2} = \sqrt{10}$

$x-6 = \pm \sqrt{10} + 6$

$x = 6 \pm \sqrt{10}$

11. A ball is thrown into the air. The height h , in feet, of the ball can be modeled by the equation

$f(t) = -16t^2 + 20t + 36$, where t is the time, in seconds, the ball is in the air.

a). How high off the ground is the ball after 2 seconds?

$$f(2) = -16(2)^2 + 20(2) + 36 = 12 \text{ feet}$$

c) At what height was the ball thrown from?

36 feet (0, 36) y-int

d) When did the ball reach its maximum height?

$$\frac{-20}{2(-16)} = 0.625 \text{ seconds}$$

b). When will the ball hit the ground?

$$0 = -16t^2 + 20t + 36$$

$$0 = -4t^2 + 5t + 9$$

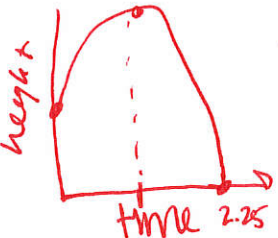
$$\frac{-5 \pm \sqrt{5^2 - 4(-4)(9)}}{2(-4)} = \frac{-5 \pm \sqrt{169}}{-8}$$

$$\frac{-5 + 13}{-8} = \frac{-5 - 13}{-8} = \frac{-8}{-8} = -1 \rightarrow 10$$

2.25 seconds

e) What is the maximum height?

$$K f(0.625) = -16(0.625)^2 + 20(0.625) + 36 = 42.25 \text{ feet}$$



12. Calculate the average rate of change of the function $y = 3x^2 + 6x - 8$ on the interval $[-2, 0]$

$$\frac{f(0) - f(-2)}{0 - (-2)} = \frac{-8 - 8}{2} = \frac{-16}{2} = -8$$

Find a value for c that will make each polynomial a perfect square trinomial.

13. $x^2 + 16x + c$ $\boxed{64}$ $\left(\frac{16}{2}\right)^2 = 8^2$
 $= (x + 8)^2$

14. $x^2 - 4x + c$ $\boxed{4}$ $\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$
 $(x - 2)^2$

15. Write the general equation of a quadratic in each of the following forms:

Standard Form:

$$y = ax^2 + bx + c$$

Vertex Form:

$$y = a(x - h)^2 + k$$

16. Given $y = x^2 + 2x - 3$, find the following:

a) Factored Form:

$$y = (x + 3)(x - 1)$$

$$\frac{-2}{2} = -1$$

b) Vertex Form:

$h = -1, k = -4, a = 1$

$$\frac{-2}{2(1)} = -\frac{2}{2} = -1$$

$$(-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4$$

$$y = (x + 1)^2 - 4$$

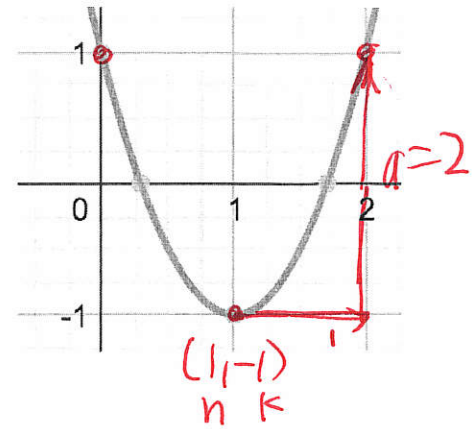
17. Write a quadratic equation for the given graph. (HINT: Vertex form)

$$y = 2(x-1)^2 - 1 \quad \text{or} \quad y = 2(x-1)(x-1) - 1$$

$$= 2(x^2 - 2x + 1) - 1$$

$$= 2x^2 - 4x + 2 - 1$$

$$y = 2x^2 - 4x + 1$$



18. Write a quadratic equation with the solutions -5 and $\frac{2}{3}$ in standard form.

$$x = -5 \quad x = \frac{2}{3}$$

$$x + 5 = 0 \quad 3x - 2 = 0$$

$$(x+5)(3x-2)$$

$$y = 3x^2 - 2x + 15x - 10$$

$$y = 3x^2 + 13x - 10$$

Simplify, no negative exponents allowed.

19. $(2a^{\frac{1}{2}}b)^3 (3a^{\frac{3}{4}}b^{\frac{1}{3}})$

$$8a^{\frac{3}{2} + \frac{3}{4}} b^3$$

$$= 24a^{\frac{9}{4}} b^{\frac{10}{3}}$$

20. $\frac{16x^2y^{\frac{1}{4}}z^{\frac{1}{8}}}{48x^{\frac{3}{4}}y^{\frac{1}{2}}z^{\frac{1}{3}}}$ $\frac{1}{4} - \frac{2}{4} = -\frac{1}{4}$

$$\frac{x^{\frac{5}{4}}}{3y^{\frac{1}{4}}}$$

21. $\frac{3 + \sqrt{45}}{6}$

$$\frac{3 + 3\sqrt{5}}{6}$$

$$= \frac{1 + \sqrt{5}}{2}$$

$$\sqrt{45} = 3\sqrt{5}$$

$$3 \cdot 3$$

22. Sketch the graph of $f(t) = -t^2 - 6t - 5$ by using the answers to parts (a-g) below:

a) Axis of Symmetry: $\frac{b}{2(-1)} = \frac{6}{-2} = -3$ $x = -3$

b) Vertex: $(-3, 4)$ $-(-3)^2 - 6(-3) - 5$
 $-9 + 18 - 5$

c) Direction of Opening:

Down

d) Minimum or Maximum:

Maximum

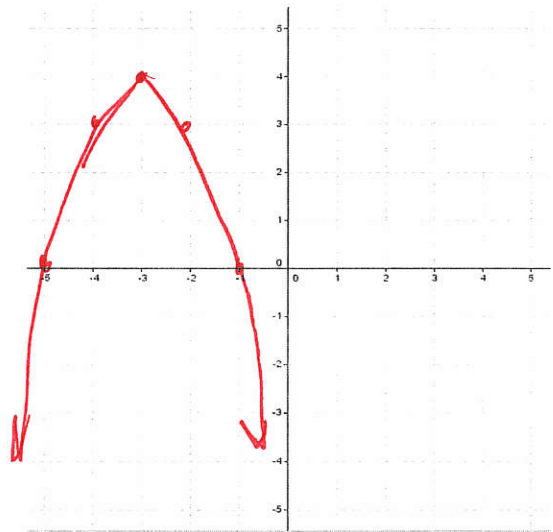
e) y-intercept:

$$(0, -5)$$

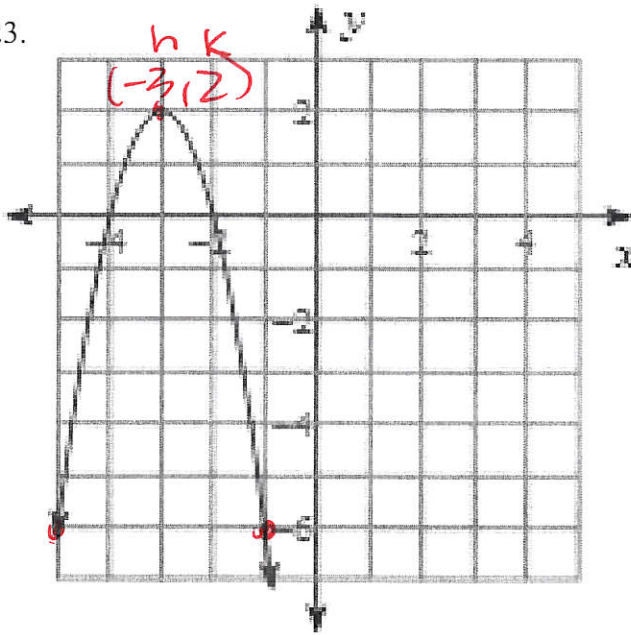
f) x-intercept(s):

$$x = -5, x = -1$$

$$(-5, 0) \quad (-1, 0)$$



23.



a) State the vertex:

$$\boxed{(-3, 2)}$$

b) Find the equation of the parabola:

$$a = -2 \quad (h, k) = (-3, 2)$$

$$\boxed{y = -2(x+3)^2 + 2} \quad \text{or} \quad \boxed{y = 2x^2 - 12x - 16}$$

c) State the zeros:

$$\boxed{x = -4 \quad x = -2}$$

d) State the y-intercept:

$$\boxed{(0, -16)}$$

$$y = -2(0+3)^2 + 2 = -2(9) + 2 = -18 + 2 = -16$$

e) State the domain:

$$\boxed{(-\infty, \infty)}$$

f) State the range:

$$\boxed{(-\infty, 2]}$$

g) Find $f(-1)$

$$y = -2(-1+3)^2 + 2 = -2(4) + 2 = -8 + 2 = -6$$

$$\boxed{(-1, -6)}$$

Find vertex form if needed. Then write a verbal expression for each equation describing the transformation from the parent function.

24. $y = -(x - 7)^2 + 3$

right 7, up 3, reflected down (across x-axis)

25. $y = x^2 - 6x + 4$

$$\frac{b}{2a} = 3$$

$$(h, k) = (3, -5) \quad 3^2 - 6(3) + 4$$

$$y = (x - 3)^2 - 5$$

right 3, down 5

26. Answer the following questions about the function that has solutions of -2 and 4.

$$(x+2)(x-4)$$

Factored form:

$$\boxed{y = (x+2)(x-4)}$$

Standard form:

$$\boxed{y = x^2 - 2x - 8}$$

Vertex form:

$$\boxed{y = (x-1)^2 - 9}$$

$$\frac{2}{2(1)} = 1 \quad 1^2 - 2(1) - 8 = -9$$

$$(h, k) = (1, -9)$$

y-intercept

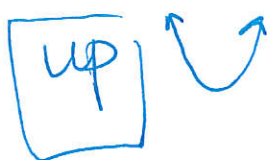
$$(0, c) = \boxed{(0, -8)}$$

Axis of symmetry:

$$\frac{-b}{2a} = \frac{2}{2(1)} = 1$$

$$\boxed{x = 1}$$

Direction of Opening



For each of the parabolas, complete the following:

- A) Find the other two equivalent forms of each equation: (Hint: standard form, vertex form, and factored form.)
- B) State or find the vertex
- C) Find the y-intercept.
- D) Find the x-intercept(s), if any. (Hint: try factoring, square root method or the quadratic formula)
- E) Graph

27. $y = -(x-2)^2 + 3$ $-(x-2)(x-2) + 3$

A) $y = -x^2 + 4x - 1$ $-(x^2 - 4x + 4) + 3$
 $-x^2 + 4x - 4 + 3$

B) $(2, 3)$

C) $(0, -1)$

D) Quadratic formula
 $a = -1, b = 4, c = -1$

E) $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

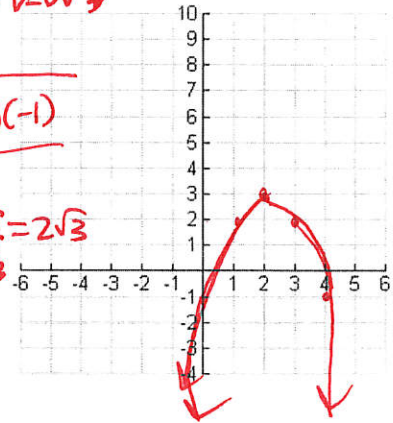
$y = -x^2 + 4x - 1$
 ~~$\frac{1}{4}$~~
 no factored form

$\frac{-4 \pm \sqrt{4^2 - 4(-1)(-1)}}{2(-1)}$

$\frac{-4 \pm \sqrt{12}}{-2}$ $\sqrt{12} = 2\sqrt{3}$

$\frac{-4 \pm 2\sqrt{3}}{-2}$

$+ 2 \pm \sqrt{3} = (3.73, 0)$
 $(-2.6, 0)$ \square



28. $y = (x+1)(x-3)$

A) $y = x^2 - 2x - 3$

B) $(1, -4)$

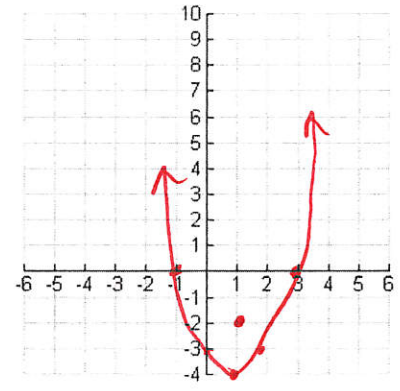
C) $(0, -3)$

D) $x = -1, x = 3$

E)

$y = x^2 - 2x + 1 - 3 + 1$
 $y = (x-1)^2 - 4$

$(-1, 0) (3, 0)$



29. Write a quadratic equation given the following: Vertex $(-1, 1)$ and a point $(2, 4)$.

$4 = a(2+1)^2 + 1$
 $4 = 9a + 1$
 $3 = 9a$ $a = \frac{1}{3}$

$y = \frac{1}{3}(x+1)^2 + 1$

30. Given $f(x) = ax^2 + bx + c$. State a value for a that makes $f(x)$ opens down and wider than: $g(x) = 2x^2 + 5x + 3$.

$a = -\frac{1}{2}, -1,$
 $-\frac{1}{4}, -\frac{1}{3}, -\frac{2}{3}$ etc...
 anything negative and less than 2

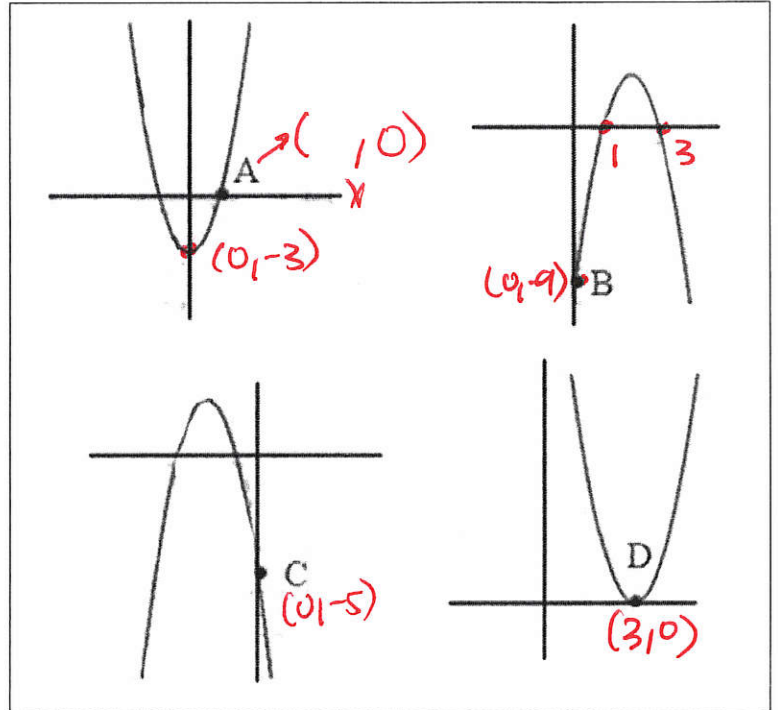
Match each equation to its corresponding graph. Then identify KEY Quadratic features that helped you determine each match.

31. C
 $y = -2x^2 - 8x - 5$
 y -int $(0, -5)$

32. B
 $x=3 \quad x=1$
 $y = (-3x + 9)(x - 1)$
 $y = -3x^2 + 2x - 9$
 y -int $(0, -9)$
 x -int $(3, 0)(1, 0)$

33. A
 $y = 2x^2 - 3$
 up \curvearrowright
 y -int $(0, -3)$
 $y = 2(x-0)^2 - 3$
 vertex $= (0, -3)$

34. D
 $y = 3(x - 3)^2 + 0$
 vertex $(3, 0)$
 up \curvearrowright



Using the equations and graphs you matched above identify the coordinates for each of the given points.

Use the equations to verify that they are accurate.

35. A $(1.2, 0)$

B $(0, -9)$

C $(0, -5)$

D $(3, 0)$

$0 = 2x^2 - 3$
 $+3$
 $3 = 2x^2$
 $\frac{3}{2} = \frac{2x^2}{2}$
 $\sqrt{\frac{3}{2}} = \sqrt{x^2}$
 $x = \pm 1.2$

$y = (0+9)(0-1)$
 $y = (9)(-1)$
 $y = -9$

$y = -2(0)^2 - 8(0) - 5$
 $y = 0 - 5$
 $y = -5$

$0 = 3(x-3)^2$
 $\frac{0}{3} = \frac{3(x-3)^2}{3}$
 $\sqrt{0} = \sqrt{(x-3)^2}$
 $0 = x-3$
 $+3$
 $3 = x$