

Applications of Quadratics - Day 1

Key

1. If an M-16 is fired straight upward, then the height $h(t)$ of the bullet in feet at time t in seconds is given by $h(t) = -16t^2 + 325t + 4$.

a) What is the starting height of the bullet?

(time, height)

4 ft

b) How long does it take for the bullet to return to the earth?

$$x = \frac{-325 \pm \sqrt{(325)^2 - 4(-16)(4)}}{2(-16)} = -0.12 \text{ and } \boxed{20.32 \text{ sec}}$$

c) What is the maximum height?

$$h = \frac{-(325)}{2(-16)} = 10.16 \quad k = -16(10.16)^2 + 325(10.16) + 4 = \boxed{1,654.39 \text{ ft}}$$

d) What is a realistic domain and range?

$$D: [0, 20.32] \quad R: [0, 1,654.39]$$

Seconds feet

e) At a height of 500 feet how much time has passed?

$$500 = -16t^2 + 325t + 4 \quad t = \frac{-325 \pm \sqrt{(325)^2 - 4(-16)(-496)}}{2(-16)} = \begin{matrix} 1.66 \text{ sec} \\ 18.65 \text{ sec} \end{matrix}$$

2. A contestant tosses a horseshoe from one pit to another with an initial vertical velocity of 50 feet per second. The horseshoe is released 3 feet above the ground. Use the model $h = -16t^2 + 50t + 3$, where h is the height (in feet) and t is the time (in seconds) to tell how long the horseshoe was in the air.

$$x = \frac{-50 \pm \sqrt{(50)^2 - 4(-16)(3)}}{2(-16)} = -0.06 \text{ and } \boxed{3.18 \text{ sec}}$$

3. The number of mosquitoes $M(x)$, in millions, in a certain area depends on the June rainfall x , in inches, according to the equation $M(x) = 10x - 2x^2$. What rainfall produces the maximum number of mosquitoes?

$$M(x) = -2x^2 + 10x \quad \frac{-10}{2(-2)} = \frac{10}{4} = \boxed{2.5 \text{ inches}}$$

4. The polynomial function $I(t) = -0.1t^2 + 1.9t$ represents the yearly income (or loss) from a real estate investment, where t is time in years after 1970. During what year does the maximum income occur?

$$h = \frac{-(1.9)}{2(-0.1)} = 9.5 \quad 1970 + 9.5 = \boxed{1979 \text{ year}}$$

5. Your company uses the quadratic model $y = -7x^2 + 350x$ to represent how many units y of a new product will be sold x weeks after its release. How many units can you expect to sell in week 27?

$$y = -7(27)^2 + 350(27) = \boxed{4,347 \text{ units}}$$

6. Your company uses the quadratic model $y = -4.5x^2 + 150x$ to represent the average number of new customers who will be signed on x weeks after the release of your new service. How many new customers can you expect to gain in week 8?

$x=8$, find y

$$y = -4.5(8)^2 + 150(8) = 912 \text{ New customers}$$

7. The profit for a company is given by $P(x) = -0.0002x^2 + 140x - 250000$, where x is the number of units produced. What production level will yield a maximum profit? (# units, profit)

Vertex
 h

$$\frac{-140}{2(-0.0002)} = 350,000 \text{ units}$$

8. A boy tosses a ball upward at 32 feet per second from a window that is 48 feet above the ground. The height of the ball above ground (in feet) at time t (in seconds) is given by $h(t) = -16t^2 + 32t + 48$.

$x=16t$

a) Find the time at which the ball strikes the ground.

$$-16t^2 + 32t + 48 = 0 \quad (t-3)(t+1) = 0 \quad 3 \text{ seconds}$$

b) At a height of 60 feet how much time as passed?

$y=60$,
find x

$$-16t^2 + 32t + 48 = 60 \quad x = \frac{-32 \pm \sqrt{(32)^2 - 4(-16)(-12)}}{2(-16)} = \frac{1}{2} \text{ or } \frac{3}{2}$$

$$-16t^2 + 32t - 12 = 0 \quad 0.5 \text{ or } 1.5 \text{ seconds}$$

9. A rock is thrown upward so that its distance, in feet, above the ground after t seconds is $h(t) = -14t^2 + 336t$

$x=16t$

a. Find the zeros of the function and explain the meaning in the context of the problem. (sec, height)

$$-14t(t-24) = 0 \quad \text{Rock is on ground at start \& again after 24 seconds}$$

$$t = 0 \quad t = 24$$

b. Find the vertex of the function and explain the meaning in the context of the problem.

$$\frac{-336}{2(-14)} = 12 \quad (12, 2016) \text{ At 12 seconds, it's 2,016 ft high}$$

c. What is a realistic Domain and Range?

$$D: [0, 24] \quad R: [0, 2016]$$

10. John owns a hotdog stand. He has found that his profit is represented by the equation $P(x) = -x^2 + 68x + 77$, with P being the profit in dollars, and x the number of hotdogs sold. How many hotdogs must he sell to earn the most profit? (hotdogs, \$)

Vertex
 h

$$\frac{-68}{2(-1)} = 34 \text{ hot dogs!}$$

11. The manufacturer of a CD player has found that the revenue R (in dollars) is $R(p) = -4p^2 + 1280p$, when the unit price is p dollars. If the manufacturer sets the price p to maximize revenue, what is the maximum revenue to the nearest whole dollar?

Vertex
 K

$$\frac{-1280}{2(-4)} = 160 \quad (160, 102,400) \quad \$102,400 \quad (\text{price, revenue})$$

$$-4(160)^2 + 1280(160) = 102,400$$