

# 3.1 Functions

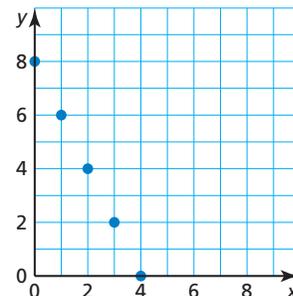
## Essential Question What is a function?

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the  $x$ -coordinates are inputs and the  $y$ -coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

### EXPLORATION 1 Describing a Function

**Work with a partner.** Functions can be described in many ways.

- by an equation
- by an input-output table
- using words
- by a graph
- as a set of ordered pairs



- Explain why the graph shown represents a function.
- Describe the function in two other ways.

### ANALYZING RELATIONSHIPS

To be proficient in math, you need to analyze relationships mathematically to draw conclusions.

### EXPLORATION 2 Identifying Functions

**Work with a partner.** Determine whether each relation represents a function. Explain your reasoning.

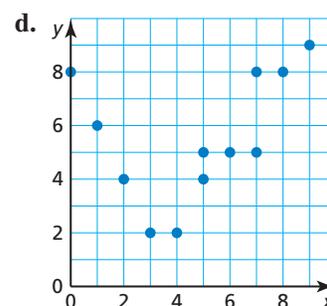
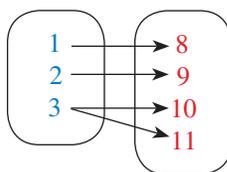
a.

Input, $x$	0	1	2	3	4
Output, $y$	8	8	8	8	8

b.

Input, $x$	8	8	8	8	8
Output, $y$	0	1	2	3	4

c. Input,  $x$       Output,  $y$



- $(-2, 5), (-1, 8), (0, 6), (1, 6), (2, 7)$
- $(-2, 0), (-1, 0), (-1, 1), (0, 1), (1, 2), (2, 2)$
- Each radio frequency  $x$  in a listening area has exactly one radio station  $y$ .
- The same television station  $x$  can be found on more than one channel  $y$ .
- $x = 2$
- $y = 2x + 3$

## Communicate Your Answer

- What is a function? Give examples of relations, other than those in Explorations 1 and 2, that (a) are functions and (b) are not functions.

# 3.1 Lesson

## Core Vocabulary

relation, p. 104  
function, p. 104  
domain, p. 106  
range, p. 106  
independent variable, p. 107  
dependent variable, p. 107

### Previous

ordered pair  
mapping diagram

## REMEMBER

A relation can be represented by a mapping diagram.



## What You Will Learn

- ▶ Determine whether relations are functions.
- ▶ Find the domain and range of a function.
- ▶ Identify the independent and dependent variables of functions.

## Determining Whether Relations Are Functions

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the  $x$ -coordinates are inputs and the  $y$ -coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

### EXAMPLE 1 Determining Whether Relations Are Functions

Determine whether each relation is a function. Explain.

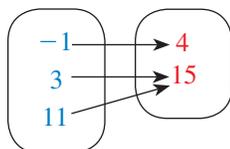
a.  $(-2, 2), (-1, 2), (0, 2), (1, 0), (2, 0)$

b.  $(4, 0), (8, 7), (6, 4), (4, 3), (5, 2)$

c.

Input, $x$	-2	-1	0	0	1	2
Output, $y$	3	4	5	6	7	8

d. **Input,  $x$**    **Output,  $y$**



## SOLUTION

- a. Every input has exactly one output.  
▶ So, the relation is a function.
- b. The input 4 has two outputs, 0 and 3.  
▶ So, the relation is *not* a function.
- c. The input 0 has two outputs, 5 and 6.  
▶ So, the relation is *not* a function.
- d. Every input has exactly one output.  
▶ So, the relation is a function.

## Monitoring Progress



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Determine whether the relation is a function. Explain.

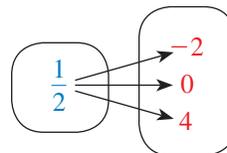
1.  $(-5, 0), (0, 0), (5, 0), (5, 10)$

2.  $(-4, 8), (-1, 2), (2, -4), (5, -10)$

3.

Input, $x$	Output, $y$
2	2.6
4	5.2
6	7.8

4. **Input,  $x$**    **Output,  $y$**

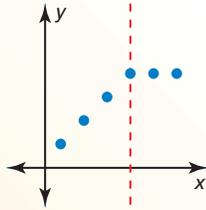


## Core Concept

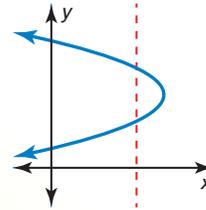
### Vertical Line Test

**Words** A graph represents a function when no vertical line passes through more than one point on the graph.

**Examples** Function



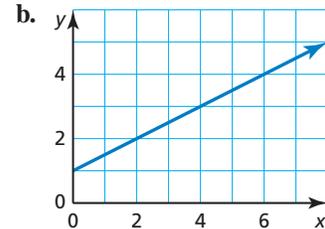
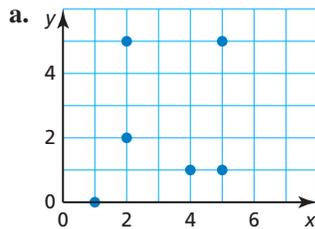
Not a function



### EXAMPLE 2

### Using the Vertical Line Test

Determine whether each graph represents a function. Explain.



### SOLUTION

a. You can draw a vertical line through (2, 2) and (2, 5).

► So, the graph does *not* represent a function.

b. No vertical line can be drawn through more than one point on the graph.

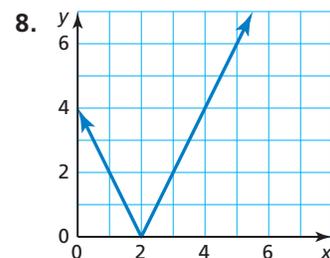
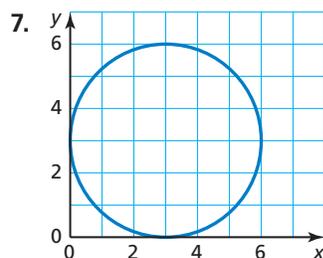
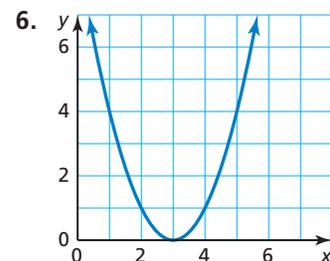
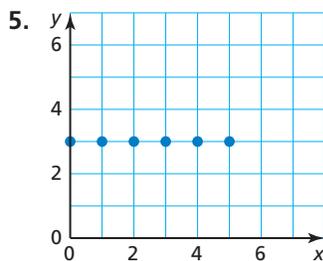
► So, the graph represents a function.

### Monitoring Progress



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Determine whether the graph represents a function. Explain.



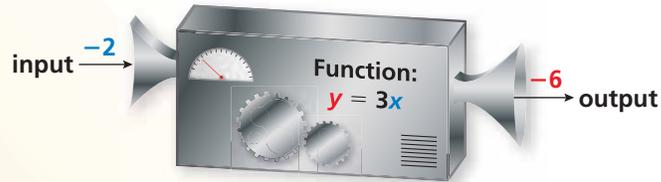
## Finding the Domain and Range of a Function

### Core Concept

#### The Domain and Range of a Function

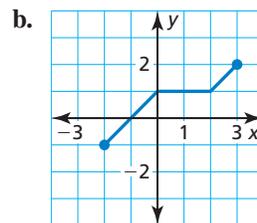
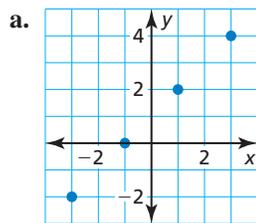
The **domain** of a function is the set of all possible input values.

The **range** of a function is the set of all possible output values.



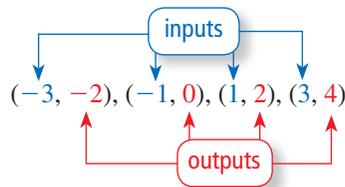
#### EXAMPLE 3 Finding the Domain and Range from a Graph

Find the domain and range of the function represented by the graph.



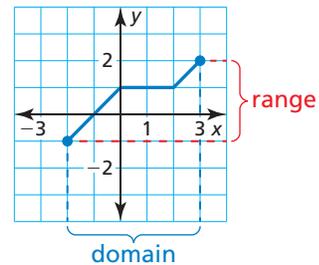
#### SOLUTION

a. Write the ordered pairs. Identify the inputs and outputs.



▶ The domain is  $-3, -1, 1,$  and  $3$ .  
The range is  $-2, 0, 2,$  and  $4$ .

b. Identify the  $x$ - and  $y$ -values represented by the graph.



▶ The domain is  $-2 \leq x \leq 3$ .  
The range is  $-1 \leq y \leq 2$ .

#### STUDY TIP

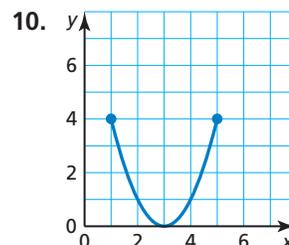
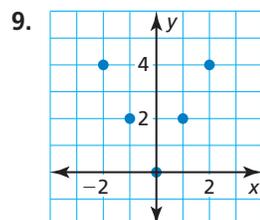
A relation also has a domain and a range.

#### Monitoring Progress



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Find the domain and range of the function represented by the graph.



## Identifying Independent and Dependent Variables

The variable that represents the input values of a function is the **independent variable** because it can be *any* value in the domain. The variable that represents the output values of a function is the **dependent variable** because it *depends* on the value of the independent variable. When an equation represents a function, the dependent variable is defined in terms of the independent variable. The statement “ $y$  is a function of  $x$ ” means that  $y$  varies depending on the value of  $x$ .

$$y = -x + 10$$

↑                      ↑  
dependent variable,  $y$       independent variable,  $x$

### EXAMPLE 4 Identifying Independent and Dependent Variables



The function  $y = -3x + 12$  represents the amount  $y$  (in fluid ounces) of juice remaining in a bottle after you take  $x$  gulps.

- a. Identify the independent and dependent variables.
- b. The domain is 0, 1, 2, 3, and 4. What is the range?

#### SOLUTION

- a. The amount  $y$  of juice remaining depends on the number  $x$  of gulps.
  - ▶ So,  $y$  is the dependent variable, and  $x$  is the independent variable.
- b. Make an input-output table to find the range.

Input, $x$	$-3x + 12$	Output, $y$
0	$-3(0) + 12$	12
1	$-3(1) + 12$	9
2	$-3(2) + 12$	6
3	$-3(3) + 12$	3
4	$-3(4) + 12$	0

- ▶ The range is 12, 9, 6, 3, and 0.

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11. The function  $a = -4b + 14$  represents the number  $a$  of avocados you have left after making  $b$  batches of guacamole.
  - a. Identify the independent and dependent variables.
  - b. The domain is 0, 1, 2, and 3. What is the range?
12. The function  $t = 19m + 65$  represents the temperature  $t$  (in degrees Fahrenheit) of an oven after preheating for  $m$  minutes.
  - a. Identify the independent and dependent variables.
  - b. A recipe calls for an oven temperature of  $350^\circ\text{F}$ . Describe the domain and range of the function.

## Vocabulary and Core Concept Check

- WRITING** How are independent variables and dependent variables different?
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

Find the range of the function represented by the table.

Find the inputs of the function represented by the table.

<b>x</b>	-1	0	1
<b>y</b>	7	5	-1

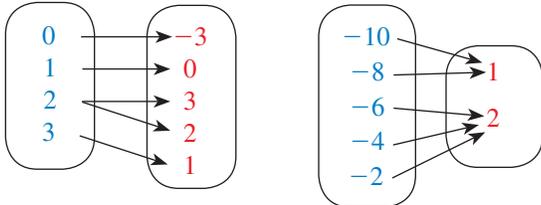
Find the  $x$ -values of the function represented by  $(-1, 7)$ ,  $(0, 5)$ , and  $(1, -1)$ .

Find the domain of the function represented by  $(-1, 7)$ ,  $(0, 5)$ , and  $(1, -1)$ .

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, determine whether the relation is a function. Explain. (See Example 1.)

- $(1, -2), (2, 1), (3, 6), (4, 13), (5, 22)$
- $(7, 4), (5, -1), (3, -8), (1, -5), (3, 6)$
- Input,  $x$**     **Output,  $y$**     6. **Input,  $x$**     **Output,  $y$**



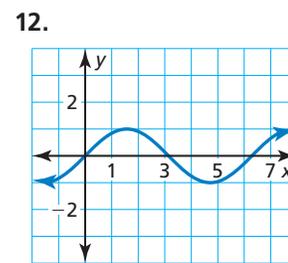
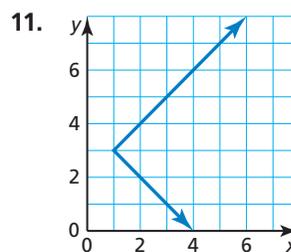
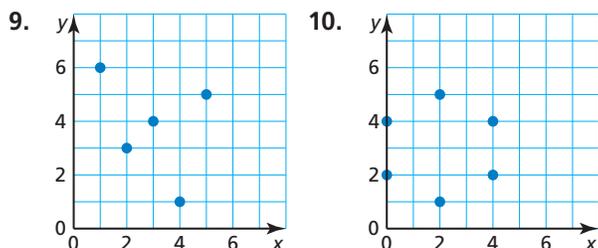
7.

<b>Input, <math>x</math></b>	16	1	0	1	16
<b>Output, <math>y</math></b>	-2	-1	0	1	2

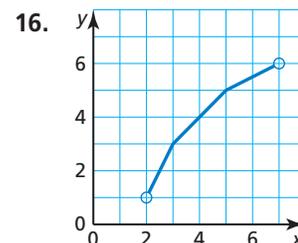
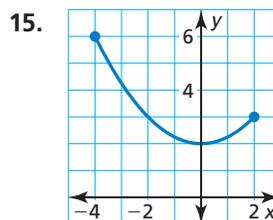
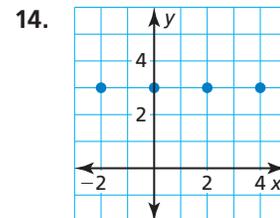
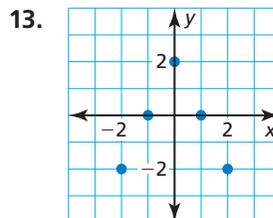
8.

<b>Input, <math>x</math></b>	-3	0	3	6	9
<b>Output, <math>y</math></b>	11	5	-1	-7	-13

In Exercises 9–12, determine whether the graph represents a function. Explain. (See Example 2.)



In Exercises 13–16, find the domain and range of the function represented by the graph. (See Example 3.)



17. **MODELING WITH MATHEMATICS** The function  $y = 25x + 500$  represents your monthly rent  $y$  (in dollars) when you pay  $x$  days late. (See Example 4.)
- Identify the independent and dependent variables.
  - The domain is 0, 1, 2, 3, 4, and 5. What is the range?

18. **MODELING WITH MATHEMATICS** The function  $y = 3.5x + 2.8$  represents the cost  $y$  (in dollars) of a taxi ride of  $x$  miles.



- Identify the independent and dependent variables.
- You have enough money to travel at most 20 miles in the taxi. Find the domain and range of the function.

**ERROR ANALYSIS** In Exercises 19 and 20, describe and correct the error in the statement about the relation shown in the table.

Input, $x$	1	2	3	4	5
Output, $y$	6	7	8	6	9

19.  The relation is *not* a function. One output is paired with two inputs.

20.  The relation is a function. The range is 1, 2, 3, 4, and 5.

**ANALYZING RELATIONSHIPS** In Exercises 21 and 22, identify the independent and dependent variables.

- The number of quarters you put into a parking meter affects the amount of time you have on the meter.
  - The battery power remaining on your MP3 player is based on the amount of time you listen to it.
23. **MULTIPLE REPRESENTATIONS** The balance  $y$  (in dollars) of your savings account is a function of the month  $x$ .

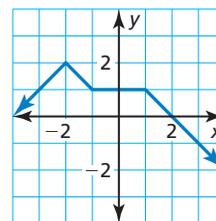
Month, $x$	0	1	2	3	4
Balance (dollars), $y$	100	125	150	175	200

- Describe this situation in words.
- Write the function as a set of ordered pairs.
- Plot the ordered pairs in a coordinate plane.

24. **MULTIPLE REPRESENTATIONS** The function  $1.5x + 0.5y = 12$  represents the number of hardcover books  $x$  and softcover books  $y$  you can buy at a used book sale.

- Solve the equation for  $y$ .
- Make an input-output table to find ordered pairs for the function.
- Plot the ordered pairs in a coordinate plane.

25. **ATTENDING TO PRECISION** The graph represents a function. Find the input value corresponding to an output of 2.



26. **OPEN-ENDED** Fill in the table so that when  $t$  is the independent variable, the relation is a function, and when  $t$  is the dependent variable, the relation is not a function.

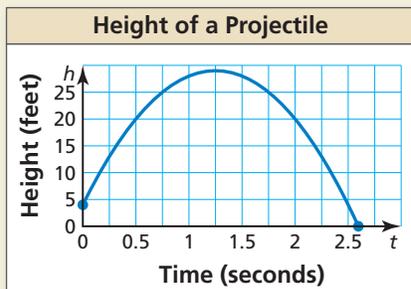
$t$				
$v$				

27. **ANALYZING RELATIONSHIPS** You select items in a vending machine by pressing one letter and then one number.



- Explain why the relation that pairs letter-number combinations with food or drink items is a function.
- Identify the independent and dependent variables.
- Find the domain and range of the function.

28. **HOW DO YOU SEE IT?** The graph represents the height  $h$  of a projectile after  $t$  seconds.



- a. Explain why  $h$  is a function of  $t$ .
- b. Approximate the height of the projectile after 0.5 second and after 1.25 seconds.
- c. Approximate the domain of the function.
- d. Is  $t$  a function of  $h$ ? Explain.
29. **MAKING AN ARGUMENT** Your friend says that a line always represents a function. Is your friend correct? Explain.

30. **THOUGHT PROVOKING** Write a function in which the inputs and/or the outputs are not numbers. Identify the independent and dependent variables. Then find the domain and range of the function.

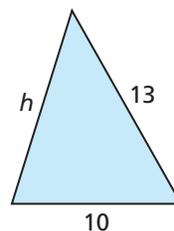
**ATTENDING TO PRECISION** In Exercises 31–34, determine whether the statement uses the word *function* in a way that is mathematically correct. Explain your reasoning.

31. The selling price of an item is a function of the cost of making the item.
32. The sales tax on a purchased item in a given state is a function of the selling price.
33. A function pairs each student in your school with a homeroom teacher.

34. A function pairs each chaperone on a school trip with 10 students.

**REASONING** In Exercises 35–38, tell whether the statement is true or false. If it is false, explain why.

35. Every function is a relation.
36. Every relation is a function.
37. When you switch the inputs and outputs of any function, the resulting relation is a function.
38. When the domain of a function has an infinite number of values, the range always has an infinite number of values.
39. **MATHEMATICAL CONNECTIONS** Consider the triangle shown.



- a. Write a function that represents the perimeter of the triangle.
- b. Identify the independent and dependent variables.
- c. Describe the domain and range of the function. (*Hint:* The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.)

**REASONING** In Exercises 40–43, find the domain and range of the function.

40.  $y = |x|$
41.  $y = -|x|$
42.  $y = |x| - 6$
43.  $y = 4 - |x|$

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the sentence as an inequality. (*Section 2.1*)

44. A number  $y$  is less than 16.
45. Three is no less than a number  $x$ .
46. Seven is at most the quotient of a number  $d$  and  $-5$ .
47. The sum of a number  $w$  and 4 is more than  $-12$ .

Evaluate the expression. (*Skills Review Handbook*)

48.  $11^2$
49.  $(-3)^4$
50.  $-5^2$
51.  $2^5$