

2.7 Dilations

Essential Question How can you enlarge or reduce a figure in the coordinate plane?

The Meaning of a Word ● Dilate

When you have your eyes checked, the optometrist sometimes

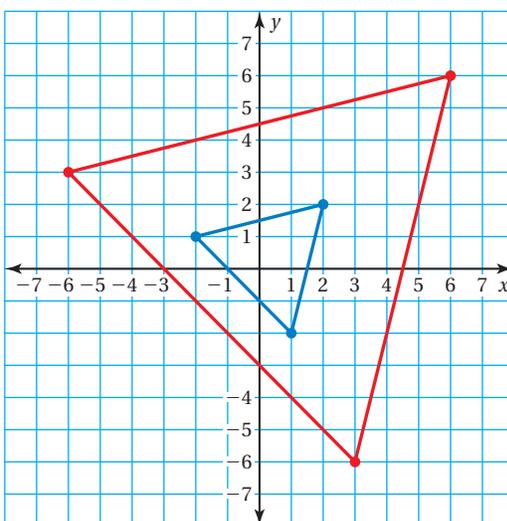
dilates one or both of the pupils of your eyes.



1 ACTIVITY: Comparing Triangles in a Coordinate Plane

Work with a partner. Write the coordinates of the vertices of the blue triangle. Then write the coordinates of the vertices of the red triangle.

- How are the two sets of coordinates related?
- How are the two triangles related? Explain your reasoning.



Geometry

In this lesson, you will

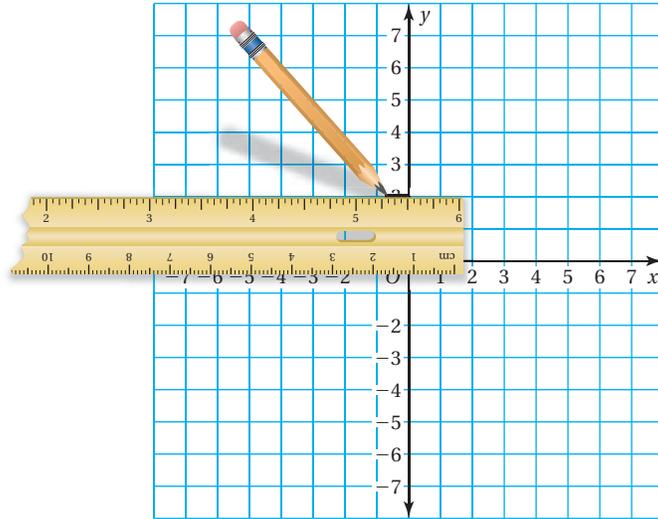
- identify dilations.
- dilate figures in the coordinate plane.
- use more than one transformation to find images of figures.

- Draw a green triangle whose coordinates are twice the values of the corresponding coordinates of the blue triangle. How are the green and blue triangles related? Explain your reasoning.
- How are the coordinates of the red and green triangles related? How are the two triangles related? Explain your reasoning.

2 ACTIVITY: Drawing Triangles in a Coordinate Plane

Work with a partner.

- Draw the triangle whose vertices are $(0, 2)$, $(-2, 2)$, and $(1, -2)$.
- Multiply each coordinate of the vertices by 2 to obtain three new vertices. Draw the triangle given by the three new vertices. How are the two triangles related?



- Repeat part (b) by multiplying by 3 instead of 2.

Math Practice

Use Prior Results

What are the four types of transformations you studied in this chapter? What information can you use to fill in your table?

3 ACTIVITY: Summarizing Transformations

Work with a partner. Make a table that summarizes the relationships between the original figure and its image for the four types of transformations you studied in this chapter.

What Is Your Answer?

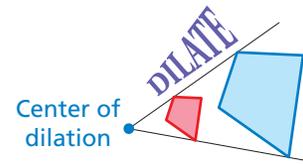
- IN YOUR OWN WORDS** How can you enlarge or reduce a figure in the coordinate plane?
- Describe how knowing how to enlarge or reduce figures in a technical drawing is important in a career such as drafting.



Practice

Use what you learned about dilations to complete Exercises 4–6 on page 87.

A **dilation** is a transformation in which a figure is made larger or smaller with respect to a point called the **center of dilation**.



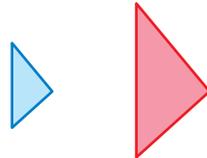
EXAMPLE 1 Identifying a Dilation

Key Vocabulary

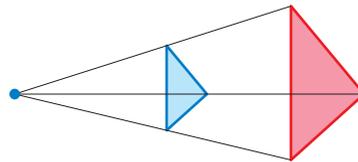
dilation, p. 84
center of dilation,
p. 84
scale factor, p. 84

Tell whether the blue figure is a dilation of the red figure.

a.

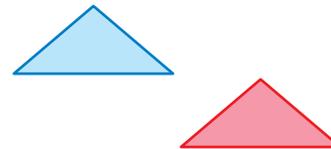


Lines connecting corresponding vertices meet at a point.



So, the blue figure is a dilation of the red figure.

b.



The figures have the same size and shape. The red figure *slides* to form the blue figure.

So, the blue figure is *not* a dilation of the red figure. It is a translation.

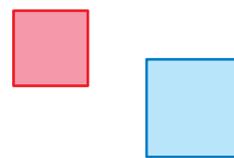
On Your Own

Tell whether the blue figure is a dilation of the red figure. Explain.

1.



2.



Now You're Ready
Exercises 7–12

In a dilation, the original figure and its image are similar. The ratio of the side lengths of the image to the corresponding side lengths of the original figure is the **scale factor** of the dilation.

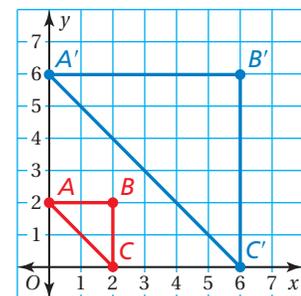
Key Idea

Dilations in the Coordinate Plane

Words To dilate a figure with respect to the origin, multiply the coordinates of each vertex by the scale factor k .

Algebra $(x, y) \rightarrow (kx, ky)$

- When $k > 1$, the dilation is an enlargement.
- When $k > 0$ and $k < 1$, the dilation is a reduction.



EXAMPLE 2 Dilating a Figure

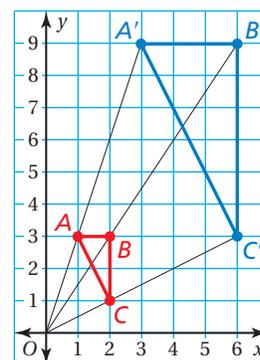
Draw the image of Triangle ABC after a dilation with a scale factor of 3. Identify the type of dilation.

Multiply each x - and y -coordinate by the scale factor 3.

Study Tip

You can check your answer by drawing a line from the origin through each vertex of the original figure. The vertices of the image should lie on these lines.

Vertices of ABC	$(3x, 3y)$	Vertices of $A'B'C'$
$A(1, 3)$	$(3 \cdot 1, 3 \cdot 3)$	$A'(3, 9)$
$B(2, 3)$	$(3 \cdot 2, 3 \cdot 3)$	$B'(6, 9)$
$C(2, 1)$	$(3 \cdot 2, 3 \cdot 1)$	$C'(6, 3)$



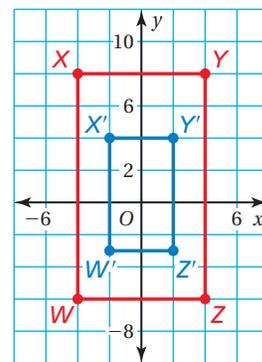
∴ The image is shown at the right. The dilation is an *enlargement* because the scale factor is greater than 1.

EXAMPLE 3 Dilating a Figure

Draw the image of Rectangle $WXYZ$ after a dilation with a scale factor of 0.5. Identify the type of dilation.

Multiply each x - and y -coordinate by the scale factor 0.5.

Vertices of $WXYZ$	$(0.5x, 0.5y)$	Vertices of $W'X'Y'Z'$
$W(-4, -6)$	$(0.5 \cdot (-4), 0.5 \cdot (-6))$	$W'(-2, -3)$
$X(-4, 8)$	$(0.5 \cdot (-4), 0.5 \cdot 8)$	$X'(-2, 4)$
$Y(4, 8)$	$(0.5 \cdot 4, 0.5 \cdot 8)$	$Y'(2, 4)$
$Z(4, -6)$	$(0.5 \cdot 4, 0.5 \cdot (-6))$	$Z'(2, -3)$



∴ The image is shown at the right. The dilation is a *reduction* because the scale factor is greater than 0 and less than 1.

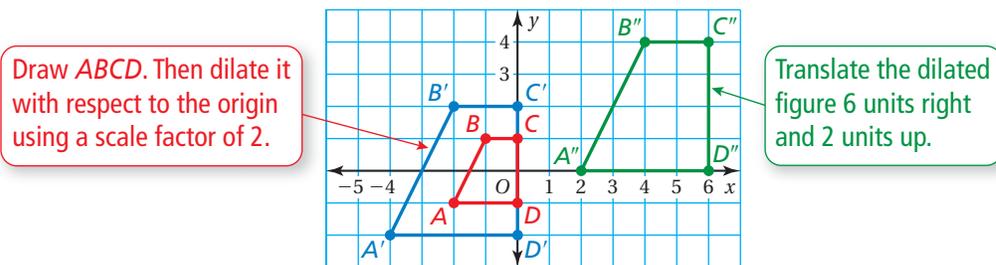
On Your Own

Now You're Ready
Exercises 13–18

- WHAT IF?** Triangle ABC in Example 2 is dilated by a scale factor of 2. What are the coordinates of the image?
- WHAT IF?** Rectangle $WXYZ$ in Example 3 is dilated by a scale factor of $\frac{1}{4}$. What are the coordinates of the image?

EXAMPLE 4 Using More than One Transformation

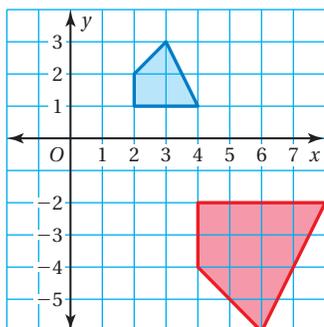
The vertices of a trapezoid are $A(-2, -1)$, $B(-1, 1)$, $C(0, 1)$, and $D(0, -1)$. Dilate the trapezoid with respect to the origin using a scale factor of 2. Then translate it 6 units right and 2 units up. What are the coordinates of the image?



- The coordinates of the image are $A''(2, 0)$, $B''(4, 4)$, $C''(6, 4)$, and $D''(6, 0)$.

The image of a translation, reflection, or rotation is congruent to the original figure, and the image of a dilation is similar to the original figure. So, two figures are similar when one can be obtained from the other by a sequence of translations, reflections, rotations, and dilations.

EXAMPLE 5 Describing a Sequence of Transformations

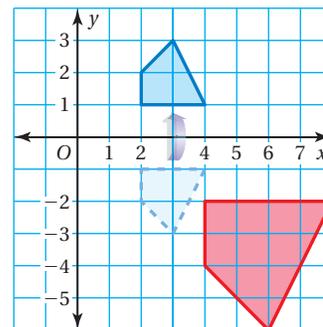


The red figure is similar to the blue figure. Describe a sequence of transformations in which the blue figure is the image of the red figure.

From the graph, you can see that the blue figure is one-half the size of the red figure. So, begin with a dilation with respect to the origin using a scale factor of $\frac{1}{2}$.

After dilating, you need to flip the figure in the x -axis.

- So, one possible sequence of transformations is a dilation with respect to the origin using a scale factor of $\frac{1}{2}$ followed by a reflection in the x -axis.



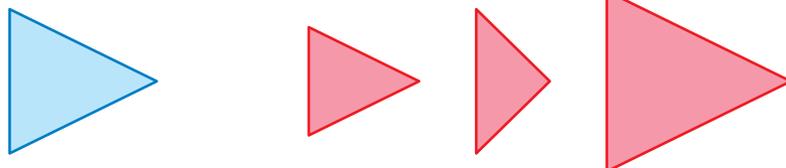
On Your Own

- In Example 4, use a scale factor of 3 in the dilation. Then rotate the figure 180° about the image of vertex C . What are the coordinates of the image?
- In Example 5, can you reflect the red figure first, and then perform the dilation to obtain the blue figure? Explain.

Now You're Ready
Exercises 23–28

Vocabulary and Concept Check

- VOCABULARY** How is a dilation different from other transformations?
- VOCABULARY** For what values of scale factor k is a dilation called an *enlargement*? a *reduction*?
- REASONING** Which figure is *not* a dilation of the blue figure? Explain.

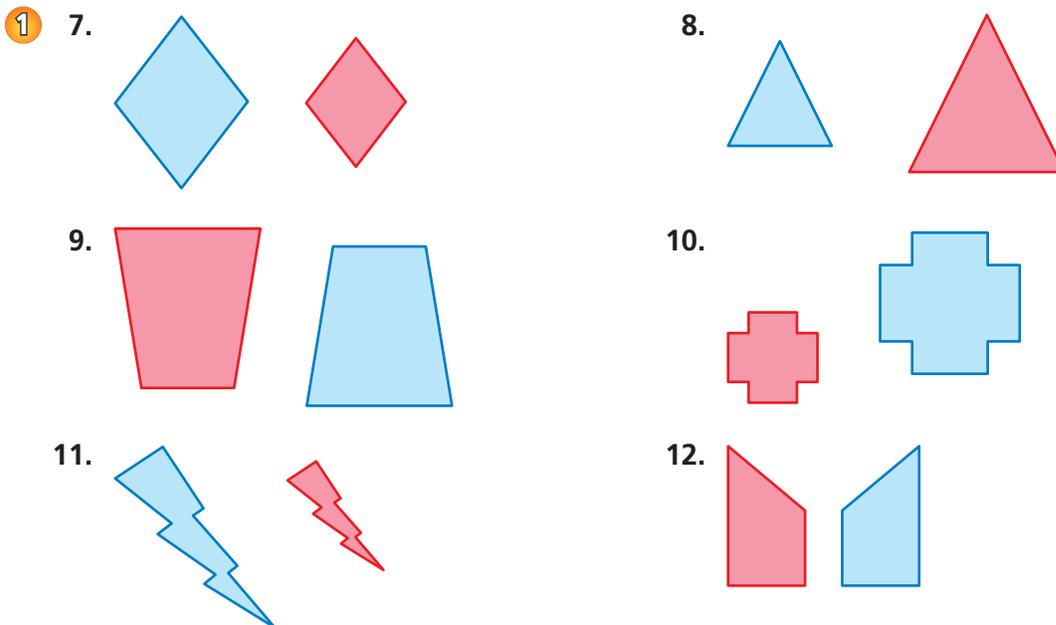


Practice and Problem Solving

Draw the triangle with the given vertices. Multiply each coordinate of the vertices by 3, and then draw the new triangle. How are the two triangles related?

4. $(0, 2), (3, 2), (3, 0)$ 5. $(-1, 1), (-1, -2), (2, -2)$ 6. $(-3, 2), (1, 2), (1, -4)$

Tell whether the blue figure is a dilation of the red figure.



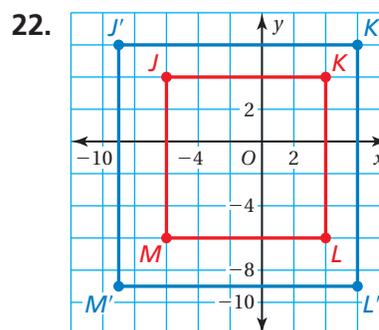
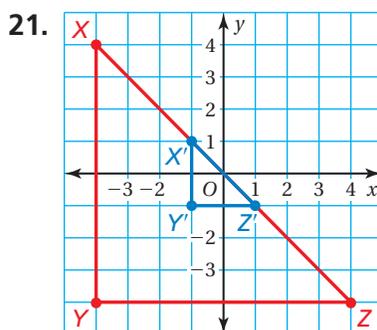
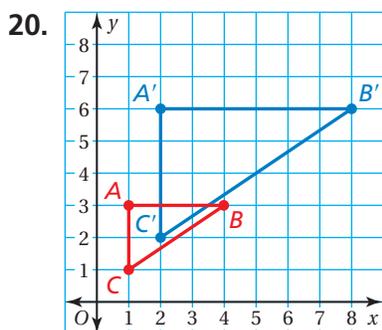
The vertices of a figure are given. Draw the figure and its image after a dilation with the given scale factor. Identify the type of dilation.

- 2 3 13. $A(1, 1), B(1, 4), C(3, 1); k = 4$ 14. $D(0, 2), E(6, 2), F(6, 4); k = 0.5$
 15. $G(-2, -2), H(-2, 6), J(2, 6); k = 0.25$ 16. $M(2, 3), N(5, 3), P(5, 1); k = 3$
 17. $Q(-3, 0), R(-3, 6), T(4, 6), U(4, 0); k = \frac{1}{3}$ 18. $V(-2, -2), W(-2, 3), X(5, 3), Y(5, -2); k = 5$

19. **ERROR ANALYSIS** Describe and correct the error in listing the coordinates of the image after a dilation with a scale factor of $\frac{1}{2}$.

Vertices of ABC	(2x, 2y)	Vertices of A'B'C'
A(2, 5)	(2 • 2, 2 • 5)	A'(4, 10)
B(2, 0)	(2 • 2, 2 • 0)	B'(4, 0)
C(4, 0)	(2 • 4, 2 • 0)	C'(8, 0)

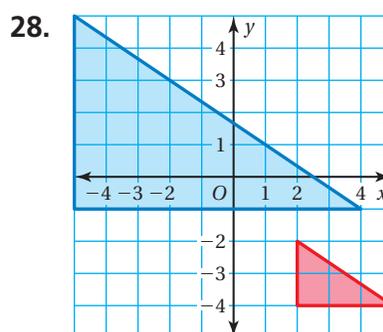
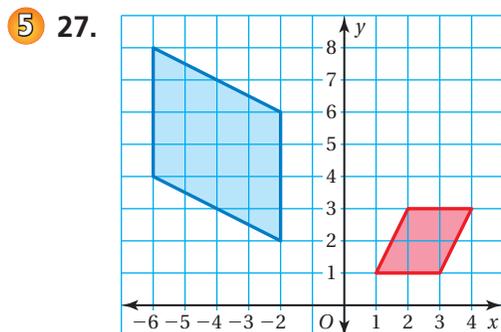
The blue figure is a dilation of the red figure. Identify the type of dilation and find the scale factor.



The vertices of a figure are given. Find the coordinates of the figure after the transformations given.

23. A(-5, 3), B(-2, 3), C(-2, 1), D(-5, 1)
Reflect in the y -axis. Then dilate with respect to the origin using a scale factor of 2.
24. F(-9, -9), G(-3, -6), H(-3, -9)
Dilate with respect to the origin using a scale factor of $\frac{2}{3}$. Then translate 6 units up.
25. J(1, 1), K(3, 4), L(5, 1)
Rotate 90° clockwise about the origin. Then dilate with respect to the origin using a scale factor of 3.
26. P(-2, 2), Q(4, 2), R(2, -6), S(-4, -6)
Dilate with respect to the origin using a scale factor of 5. Then dilate with respect to the origin using a scale factor of 0.5.

The red figure is similar to the blue figure. Describe a sequence of transformations in which the blue figure is the image of the red figure.

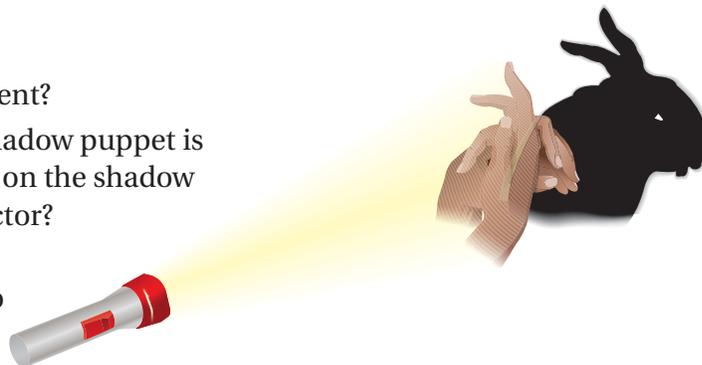


29. **STRUCTURE** In Exercises 27 and 28, is the blue figure still the image of the red figure when you perform the sequence in the opposite order? Explain.

30. **OPEN-ENDED** Draw a rectangle on a coordinate plane. Choose a scale factor of 2, 3, 4, or 5, and then dilate the rectangle. How many times greater is the area of the image than the area of the original rectangle?

31. **SHADOW PUPPET** You can use a flashlight and a shadow puppet (your hands) to project shadows on the wall.

- Identify the type of dilation.
- What does the flashlight represent?
- The length of the ears on the shadow puppet is 3 inches. The length of the ears on the shadow is 4 inches. What is the scale factor?
- Describe what happens as the shadow puppet moves closer to the flashlight. How does this affect the scale factor?



32. **REASONING** A triangle is dilated using a scale factor of 3. The image is then dilated using a scale factor of $\frac{1}{2}$. What scale factor could you use to dilate the original triangle to get the final image? Explain.

CRITICAL THINKING The coordinate notation shows how the coordinates of a figure are related to the coordinates of its image after transformations. What are the transformations? Are the figure and its image similar or congruent? Explain.

33. $(x, y) \rightarrow (2x + 4, 2y - 3)$ 34. $(x, y) \rightarrow (-x - 1, y - 2)$ 35. $(x, y) \rightarrow \left(\frac{1}{3}x, -\frac{1}{3}y\right)$

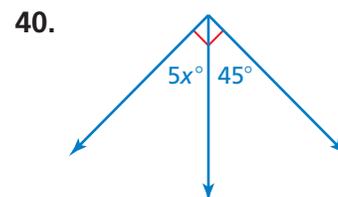
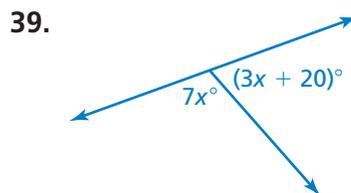
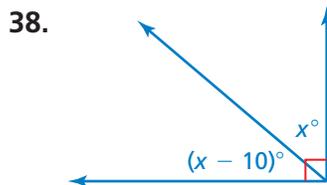
36. **STRUCTURE** How are the transformations $(2x + 3, 2y - 1)$ and $(2(x + 3), 2(y + 1))$ different?

37. **Problem Solving** The vertices of a trapezoid are $A(-2, 3)$, $B(2, 3)$, $C(5, -2)$, and $D(-2, -2)$. Dilate the trapezoid with respect to vertex A using a scale factor of 2. What are the coordinates of the image? Explain the method you used.



Fair Game Review what you learned in previous grades & lessons

Tell whether the angles are *complementary* or *supplementary*. Then find the value of x . *(Skills Review Handbook)*



41. **MULTIPLE CHOICE** Which quadrilateral is *not* a parallelogram? *(Skills Review Handbook)*

- (A) rhombus (B) trapezoid (C) square (D) rectangle