Bell Ringer

Wednesday 2/5

Using the matrices below, find each product, sum or difference.

$E = \begin{bmatrix} 1 & -3 \\ 6 & 0 \end{bmatrix}$ 2x2 $\quad F = \begin{bmatrix} 2 & 4 \\ 7 & -2 \end{bmatrix}$ 2x2 $\quad G = \begin{bmatrix} 2 & 8 & 1 \\ -1 & 3 & 3 \end{bmatrix}$ 2x3 $\quad H = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ 3x1

1. $3E$ 2. $2F$ 3. EF 4. GH 5. EH

$\begin{bmatrix} 3 & -9 \\ 18 & 0 \end{bmatrix}$ $\begin{bmatrix} 4 & 8 \\ 14 & -4 \end{bmatrix}$ $\begin{bmatrix} 2+16+3 \\ -1+6+9 \end{bmatrix}$ $\begin{bmatrix} 21 \\ 14 \end{bmatrix}$ NOT possible

$(2-21)$ $(4+6)$

$(12+0)$ $(24+0)$

$\begin{bmatrix} -19 & 10 \\ 12 & 24 \end{bmatrix}$

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Questions???

Multiplying Matrices Name: ______________________ Hour________

Examples:

1. $\begin{bmatrix} 2 & 5 & x \\ 3 & 7 & 2y \end{bmatrix} + \begin{bmatrix} 2 & 5 & x \\ 3 & 7 & 2y \end{bmatrix} + \begin{bmatrix} 2 & 5 & x \\ 3 & 7 & 2y \end{bmatrix} =$

2. How many times did you add that matrix to itself? ______

3. Rewrite problem #1 as a *multiplication problem.*

4. Write the matrix $\frac{9}{10}$[S] and find the answer, where [S] = $\begin{bmatrix} 10 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$

5. $-2\begin{bmatrix} 4 & -1 \\ 1 & 0 \\ 2 & 7 \end{bmatrix} =$

6. $4\begin{bmatrix} -2 & -8 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 8 \\ 6 & -5 \end{bmatrix} =$

7. $\frac{1}{4}\begin{bmatrix} 4 & 16 & 8 \\ 24 & 4 & 12 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 12 & 3 & 9 \\ 9 & 6 & 15 \end{bmatrix}$

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Multiply:

8. $\frac{1}{2}\begin{bmatrix}2 & 6 & 4\\ 9 & n & 8\end{bmatrix}$

9. $3\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}$

10. $3\begin{bmatrix}2 & 3 & 4\end{bmatrix}\begin{bmatrix}5\\ -6\\ 7\end{bmatrix}$

11. $4\begin{bmatrix}2 & 2 & 2 & 2\end{bmatrix}\begin{bmatrix}2\\ 4\\ -1\end{bmatrix}$

12. $\begin{bmatrix}3 & 6 & 7\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}$

13. $\begin{bmatrix}3 & -1 & 2 & 6\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}$

Solve for x:

14. $2\begin{bmatrix}7 & x & 3\end{bmatrix}\begin{bmatrix}x\\ x\\ 5\end{bmatrix} = \begin{bmatrix}6\end{bmatrix}$

15. $x\begin{bmatrix}2 & 3 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}$

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Correct Adding and Subtracting Matrices

Name KEY Date ____________ Period ___

Adding and Subtracting Matrices

Find each sum or difference.

1. $\begin{bmatrix}4 & 4\\ 3 & -5\end{bmatrix}+\begin{bmatrix}-3 & 1\\ -2 & 5\end{bmatrix} = \begin{bmatrix}1 & 5\\ 1 & 0\end{bmatrix}$

2. $\begin{bmatrix}4 & -2\\ 2 & 3\\ -4 & 3\end{bmatrix}-\begin{bmatrix}0 & -3\\ 3 & 1\\ 3 & -2\end{bmatrix} = \begin{bmatrix}4 & -5\\ -1 & 4\\ -1 & 1\end{bmatrix}$

3. $\begin{bmatrix}4 & 2\\ 6 & -2\\ 3 & 4\\ -3 & 0\end{bmatrix}+\begin{bmatrix}3 & -2\\ 2 & 0\\ 1 & 1\\ 1 & -2\end{bmatrix} = \begin{bmatrix}7 & 0\\ 8 & -2\\ 4 & 5\\ -2 & -2\end{bmatrix}$

4. $\begin{bmatrix}0.8 & -0.3\\ 1.7 & 2.4\end{bmatrix}-\begin{bmatrix}0.2 & 0.3\\ 0.4 & -1.4\end{bmatrix} = \begin{bmatrix}0.6 & -0.6\\ 1.3 & 3.8\end{bmatrix}$

Solve each matrix equation.

5. $C+\begin{bmatrix}3 & 5\\ 10 & 2\\ 1 & 5\end{bmatrix}=\begin{bmatrix}5 & 3\\ 8 & 12\\ 0 & 2\end{bmatrix}$ $C = \begin{bmatrix}2 & -2\\ -2 & 10\\ -1 & -3\end{bmatrix}$

6. $D-\begin{bmatrix}0 & -7 & 4\\ 9 & -1 & 5\\ -1 & 0 & -5\end{bmatrix}=\begin{bmatrix}2 & 9 & -3\\ 0 & 1 & -6\\ 4 & -1 & 12\end{bmatrix}$ $D = \begin{bmatrix}2 & 2 & 1\\ 9 & 0 & -1\\ 3 & -1 & 7\end{bmatrix}$

7. $\begin{bmatrix}2 & 3\\ -2 & -2\end{bmatrix}=X-\begin{bmatrix}-1 & -3\\ -1 & 2\end{bmatrix}$ $X = \begin{bmatrix}1 & 0\\ -3 & 0\end{bmatrix}$

8. $\begin{bmatrix}2 & 2 & 0\\ 1 & -1 & -1\end{bmatrix}=\begin{bmatrix}2 & -2 & 5\\ -3 & 3 & 2\end{bmatrix}+Y$ $Y = \begin{bmatrix}0 & 4 & -5\\ 4 & -4 & -3\end{bmatrix}$

Find each sum.

9. $\begin{bmatrix}2 & -3 & 4\\ 5 & -4 & 0\end{bmatrix}+\begin{bmatrix}-2 & 3 & -4\\ -5 & 4 & 0\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

10. $\begin{bmatrix}10 & -2\\ 4 & -1\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} = \begin{bmatrix}10 & -2\\ 4 & -1\end{bmatrix}$

Find the value of each variable.

11. $\begin{bmatrix}14 & 10\\ -7 & -1\end{bmatrix}=\begin{bmatrix}3a-1 & 2a\\ 5b+3 & a+3b\end{bmatrix}$ $a = 15$, $b = -2$

12. $\begin{bmatrix}4 & -3\\ 6 & 1\end{bmatrix}-\begin{bmatrix}2 & 0\\ -1 & -2\end{bmatrix}=\begin{bmatrix}p & q\\ 7 & r\end{bmatrix}$ $p = 2$, $q = -3$, $r = 3$

Find each matrix sum or difference if possible. If not possible, explain.

$M=\begin{bmatrix}0 & 2 & 5\\ 7 & 3 & 4\end{bmatrix}$ $N=\begin{bmatrix}-1 & -3 & 4\\ 8 & 5 & 0\end{bmatrix}$ $P=\begin{bmatrix}2 & -3 & 0\\ 8 & 2 & 4\\ 6 & -4 & 1\end{bmatrix}$ $Q=\begin{bmatrix}1 & -2 & 4\\ 4 & 6 & 2\\ 0 & -2 & 5\end{bmatrix}$

13. $M+N$ $\begin{bmatrix}-1 & -1 & 9\\ 15 & 8 & 4\end{bmatrix}$

14. $Q-P$ $\begin{bmatrix}-1 & 1 & 4\\ -4 & 4 & -2\\ -6 & 2 & 4\end{bmatrix}$

15. $Q+N$ [X] Not possible; Dif. Dimensions

16. $P+Q$ $\begin{bmatrix}3 & -5 & 4\\ 12 & 8 & 6\\ 6 & -6 & 6\end{bmatrix}$

Club Membership

	1971-1972		2010-2011	
	Males	Females	Males	Females
Book	7	27	56	58
Spanish	43	64	76	82
Chess	28	0	35	26
French	16	18	59	73

A B C D

109 239

a. Write four 4 × 1 matrices, A, B, C, and D, to represent the male and female club membership for 1971–1972 and 2010–2011.

b. Write and solve a matrix equation to find matrix X, the total number of members in each club for 1971–1972.

$$\begin{bmatrix} 34 \\ 107 \\ 28 \\ 34 \end{bmatrix}$$

c. Did the total number of female club member's increase or decrease between the two school years, and by what amount?

increase

yes, by 130

18. **Think about it:** Let $C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$, $D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$, and $C + D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

If $c_{11} \cdot d_{11} = -6$ and $c_{11} > 0$, what is the value of d_{11}?

$c_{11} \cdot d_{11} = -6 \quad c_{11} > 0 \;\&\; C + D = 0$

$d_{11} = -\sqrt{6}$

$\sqrt{6} + -\sqrt{6} = 0$

$\sqrt{6} \cdot -\sqrt{6} = -\sqrt{36} = -6$

Relay Race Results:

19. The table shows the time each member of two relay teams took to complete his leg of a relay race. Team II won the race by 3 seconds. How many seconds did Gino take to run his leg of the race?

Leg	Team I Name	Team I Time (s)	Team II Name	Team II Time (s)
1	Juan	22	Miguel	23
2	Julio	25	James	22
3	Alex	23	Gino	[illegible]
4	Ted	21	Cody	20

23

Writing: Determine whether the two matrices in each pair are equal. Explain.

20. $\begin{bmatrix} 2 & \frac{3}{4} & -1 \\ \sqrt{16} & 4 & 9 \end{bmatrix}$; $\begin{bmatrix} \frac{4}{2} & 3 & 4^2 \end{bmatrix}$

No, not the same dimension and #'s aren't equal

21. $\begin{bmatrix} 2\sqrt{9} & 3^2 \\ 7 & \frac{15}{3} \end{bmatrix}$; $\begin{bmatrix} 6 & 9 \\ 7 & 5 \end{bmatrix}$

yes, each value is equivalent to the corresponding element in the other matrix.

What is the dimension of these Matrices?

$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ 1x3

$\begin{bmatrix} 3 & 1 & 2 \\ 4 & 5 & 3 \end{bmatrix}$ 2x3

$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ 3x3

$\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ 3x1

$\begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 0 & 5 \end{bmatrix}$ 3x2

$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ 2x2

$\begin{bmatrix} 1 & 7 & 5 \\ 8 & 5 & 4 \end{bmatrix}$ 2x3

$\begin{bmatrix} 10 & 7 \\ 6 & 8 \end{bmatrix}$ 2x2

Can we multiply any of them?

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Multiply:

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3+16+35 \\ 6+20+40 \\ 9+24+45 \end{bmatrix}$$

3x3 3x1

$$\begin{bmatrix} 54 \\ 66 \\ 78 \end{bmatrix}$$

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Determinants of Matrices

A value called the determinant of A, that we denote by

$det(A)$ or $|A|$,

corresponds to every square matrix A. We will avoid the formal definition and just concentrate on its calculation for now. 2x2 3x3 4x4

Determinants are helpful in finding the inverses of square matrices. If a determinant is zero there is NO inverse.

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ONLY SQUARE MATRICES HAVE DETERMINANTS

What is a square matrix? A matrix with the same number of rows and columns is considered a square matrix

In a given matrix with dimensions ***mxn****, where* ***m*** *is the number of rows and* ***n*** *is the number of columns, when* ***m=n*** *the matrix is said to be square*

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

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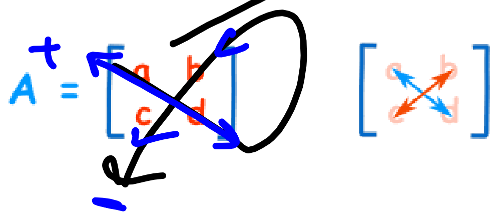
To find the determinant of A:

$|A| = ad - bc$



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Find det(A) I do...

$$A = \begin{bmatrix} 5 & 3 \\ -1 & 4 \end{bmatrix}$$

20 − (−3)

= 23

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Find det(B) We do...

$$B = \begin{bmatrix} 4 & 9 \\ 3 & 7 \end{bmatrix}$$

28 − 27

1

Find |C|

You do...

$$C = \begin{bmatrix} -6 & 3 \\ 1 & 2 \end{bmatrix}$$

-12 - 3

= -15

Find |D|

You do...

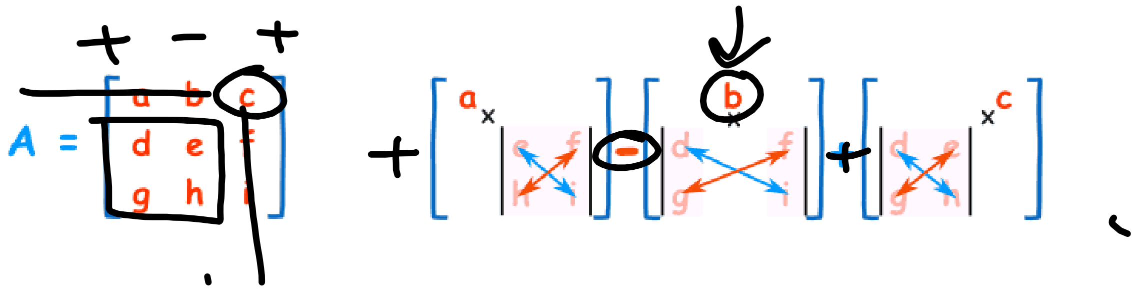
$$D = \begin{bmatrix} 4 & -2 \\ 6 & 3 \end{bmatrix}$$

12 - (-12)

= 24

For a 3x3 (and larger)

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$



Expansion by minors

$$|A| = a_{11}|M_{11}| - a_{12}|M_{12}| + a_{13}|M_{13}|$$

$$= a_{11}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} - a_{12}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} + a_{13}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

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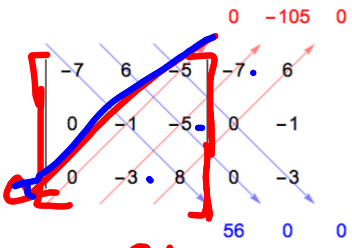

$56 + 0 + 0 - (0 - 105 + 0]$

$\therefore \begin{vmatrix} -7 & 6 & -5 \\ 0 & -1 & -5 \\ 0 & -3 & 8 \end{vmatrix} = 56 + 0 + 0 - (0 + -105 + 0) = 161$

$\boxed{161}$

Determinant using Diagonals

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Find |A|

I do...

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

$1 \cdot (18 - 0)$

$- [2(30 - 0)]$

$+ 3(25 - 12)$

$18 - 60 + 39 = \boxed{-3}$

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Find det(B) We do...

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 5 & 6 \end{bmatrix}$$

$1 \cdot (12 - 5)$

$-2(18 - 4)$

$+3(15 - 8)$

$= 7 - 28 + 21 = \boxed{0}$

Find det(C) You do...

$$C = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & 5 \\ 1 & 0 & 7 \end{bmatrix} \begin{matrix} 2 & 1 \\ 0 & 2 \\ 1 & 0 \end{matrix}$$

$= (28 + 5 + 0) - (8 + 0 + 0)$

$33 - 8 = \boxed{25}$

Due Friday

Name________________________________ Hour 1. 2. 3. 4. 5

Determinants

Find the value of each determinant.

1. $\begin{vmatrix} 10 & 6 \\ 5 & 5 \end{vmatrix}$ 2. $\begin{vmatrix} 8 & 5 \\ 6 & 1 \end{vmatrix}$ 3. $\begin{vmatrix} -7 & 3 \\ -9 & 7 \end{vmatrix}$

4. $\begin{vmatrix} -2 & 4 \\ 3 & -6 \end{vmatrix}$ 5. $\begin{vmatrix} 2 & -7 \\ -5 & 3 \end{vmatrix}$ 6. $\begin{vmatrix} -6 & -2 \\ 8 & 5 \end{vmatrix}$

7. $\begin{vmatrix} -9 & 0 \\ -12 & -7 \end{vmatrix}$ 8. $\begin{vmatrix} 6 & 14 \\ -3 & -8 \end{vmatrix}$ 9. $\begin{vmatrix} 15 & 11 \\ 23 & 19 \end{vmatrix}$

10. $\begin{vmatrix} 21 & 43 \\ 16 & 31 \end{vmatrix}$

Evaluate each determinant using expansion by minors.

11. $\begin{vmatrix} 3 & 1 & 2 \\ 0 & 6 & 4 \\ 2 & 5 & 1 \end{vmatrix}$ 12. $\begin{vmatrix} 7 & 3 & -4 \\ -2 & 9 & 6 \\ 0 & 0 & 0 \end{vmatrix}$ 13. $\begin{vmatrix} -2 & 7 & -2 \\ 4 & 5 & 2 \\ 1 & 0 & -1 \end{vmatrix}$

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Evaluate each determinant using diagonals.

14. $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 9 & 5 \\ 8 & 7 & 4 \end{vmatrix}$ 15. $\begin{vmatrix} 1 & 5 & 2 \\ -6 & -7 & 8 \\ 5 & 9 & -3 \end{vmatrix}$ 16. $\begin{vmatrix} 8 & -9 & 0 \\ 1 & 5 & 4 \\ 6 & -2 & 3 \end{vmatrix}$

17. Solve for x if det$\begin{bmatrix} 2 & x \\ 5 & -3 \end{bmatrix}$ = 24

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