Key

Applications of Quadratics - Day 2

1. A firework is shot upward so that its distance, in feet, above the ground after t seconds is (sec, height) $h(t) = -13t^2 + 312t$.

x-int

a. Find the zeros of the function and explain the meaning in the context of the problem.

$-13t(t^2-24)$

$-13t=0 \quad t-24=0$

$t = 0$ and 24

Shoots at 0 seconds and lands at 24 seconds

b. Find the vertex of the function and explain the meaning in the context of the problem.

$\frac{-(312)}{2(-13)} = 12$

$h(12) = -13(12)^2 + 312(12)$
$= 1{,}872$

At 12 seconds it reaches a max height of 1,872 ft

2. From 1970-1990, the average cost of a new car C (in dollars) can be approximated by the model $C = 30.5t^2 + 4192$, where t is the number of years since 1970. During which year was the average cost of a new car \$7,242?

y = 7242 find x

$30.5t^2 + 4192 = 7242$
$-7242 \quad -7242$

$\frac{30.5t^2}{30.5} = \frac{3{,}050}{30.5}$

$t^2 = 100$

$t = \pm 10$

$1970 + 10 = \boxed{1980}$ (year)

3. The height $h(x)$ (in feet) of a ball thrown by a child is $h(x) = -\frac{1}{12}x^2 + x + 2$ where x is the horizontal distance (in feet) from where the ball is thrown.

(horizontal distance, vertical height)

vertex, k

a. How high is the ball when it is at its maximum height?

(6, 5) h k

$h = \frac{-1}{2(-\frac{1}{12})} = \frac{-1}{-\frac{1}{6}} = 6$

$k = -\frac{1}{12}(6)^2 + 6 + 2$

$\boxed{5 \text{ ft}}$

y-int

b. How high is the ball when it leaves the child's hand?

$\boxed{2 \text{ ft}}$

x-int

c. How far from the child does the ball strike the ground?

$x = \frac{-1 \pm \sqrt{1^2 - 4(-\frac{1}{12})(2)}}{2(-\frac{1}{12})} = -1.75 \ \&\ \boxed{13.5 \text{ ft}}$

d. What is a realistic domain and range?

D: [0, 13.5] R: [0, 5]

y=4 find x

e. At a height of 4 feet how far has the ball gone?

$-\frac{1}{12}x^2 + x + 2 = 4$
$-4 \quad -4$

$-\frac{1}{12}x^2 + x - 2 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(-\frac{1}{12})(-2)}}{2(-\frac{1}{12})} = $ 2.54 ft & 9.46 ft

4. A bottle rocket is fired from the ground upwards at 64 feet per second. Using the quadratic model $h(t) = -16t^2 + 64t$ find the following:

(time, height)
(sec) (ft)

vertex k

a. What is the maximum height the bottle rocket reaches?

$h = \frac{-64}{2(-16)} = 2$ $-16(2)^2 + 64(2) = \boxed{64 \text{ ft}}$

x-int

b. How long does it take for the bottle rocket to hit the ground?

$-16t(t-4) = 0$
$t = 0,\ t = 4$

$\boxed{4 \text{ seconds}}$

starts on ground at 0 seconds...

5. Suppose the cost of producing x crates of pencils is given by $C(x) = \frac{1}{2}x^2 - 10x + 1000$. Find the following:

(crates, cost)

a. How much does it cost to produce 100 crates of pencils?

x = 10

$\frac{1}{2}(10)^2 - 10(10) + 1000 = \boxed{\$5,000}$

vertex h

b. How many crates of pencils will minimize the cost of production?

$\frac{-(-10)}{2(\frac{1}{2})} = \frac{10}{1} = \boxed{10 \text{ crates}}$

6. A geyser sends a blast of boiling water high into the air. During the eruption, the height *h* (in feet) of the water *t* seconds after being forced out from the ground can be modeled by $h = -16t^2 + 70t$. How long is the boiling water in the air?

x-int

(seconds, height)

$-2t(8t + 35) = 0$

$-2t = 0$ $8t + 35 = 0$

$t = 0$ $t = +35/8 \approx 4.38$

$\boxed{4.38 \text{ seconds}}$

7. A projectile is thrown upward so that its distance above the ground after t seconds is $h(t) = -12t^2 + 504t$. What is the maximum height of the projectile? (time, height)

vertex k

(sec, height)

$h = \frac{-504}{2(-12)} = 21$ $k = -12(21)^2 + 504(21) = \boxed{5,292 \text{ ft}}$

8. When an object is dropped, its height in feet, *h*, can be determined after *t* seconds by using the falling object model $h = -16t^2 + s$, where *s* is the initial height in feet. Find the time it takes an object to hit the ground when it is dropped from a height of 196 feet.

(seconds, height)

Find t when h=0 (s=196)

$h = -16t^2 + 196$

$0 = -16t^2 + 196$

$\frac{-16t^2}{-16} = \frac{-196}{-16}$

$\sqrt{t^2} = \sqrt{12.25}$

$t = \pm 3.5$

$\boxed{3.5 \text{ seconds}}$

9. Find an expression that could represent the length and the width of a billboard given the area of the billboard is $A = x^2 + 14x + 48$.

48
6 8

$A = (x+6)(x+8)$

$\boxed{(x+6) \text{ or } (x+8)}$