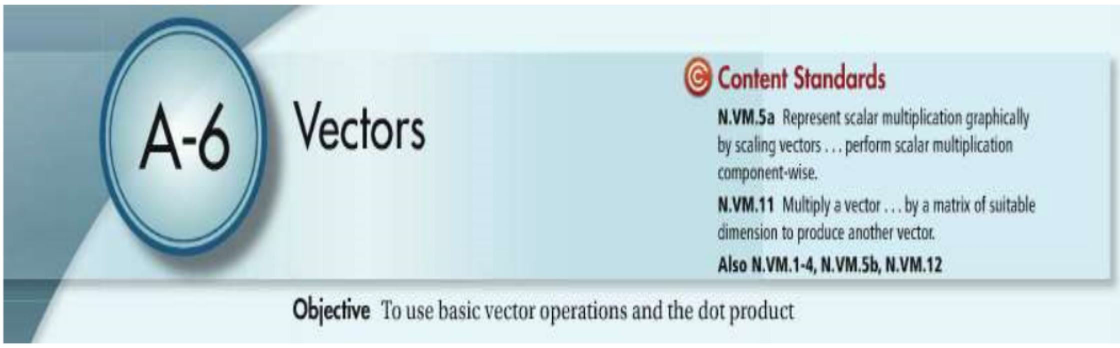
Get out old books! Or a notebook to write on :)

pg H47


A-6 Vectors

Content Standards

N.VM.5a Represent scalar multiplication graphically by scaling vectors . . . perform scalar multiplication component-wise.

N.VM.11 Multiply a vector . . . by a matrix of suitable dimension to produce another vector.

Also N.VM.1-4, N.VM.5b, N.VM.12

Objective To use basic vector operations and the dot product

PH47

Essential Understanding A *vector* is a mathematical object that has both *magnitude* (size) and direction.

Video pg H47

take note

Key Concept Vectors in Two Dimensions

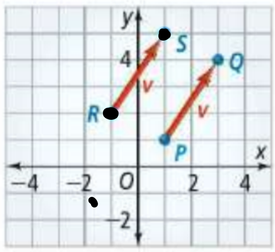
A vector has magnitude and direction. You can describe a vector as a directed line segment with initial and terminal points. Two such segments with the same magnitude and direction represent the same vector.

$\mathbf{v} = \overrightarrow{PQ}$ where $P = (1, 1)$ and $Q = (3, 4)$ and

$\mathbf{v} = \overrightarrow{RS}$ where $R = (-1, 2)$ and $S = (1, 5)$

represent the same vector.

$\|v\|$

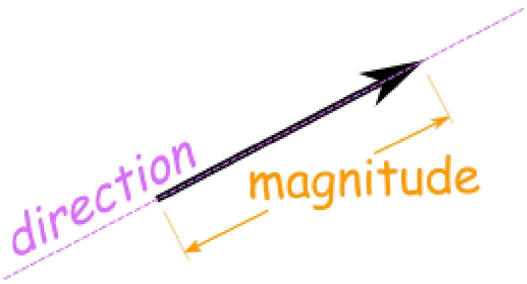


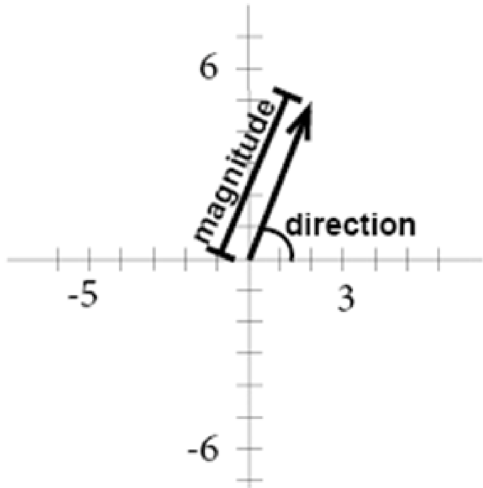
A **vector** has both magnitude and direction. You often use an arrow to represent a vector. The **magnitude** of a vector **v** is the length of the arrow. You can denote it as $|\mathbf{v}|$. You show the direction of the vector by the **initial point** and the **terminal point** of the arrow.

PH47

The magnitude is the __length__

Vectors

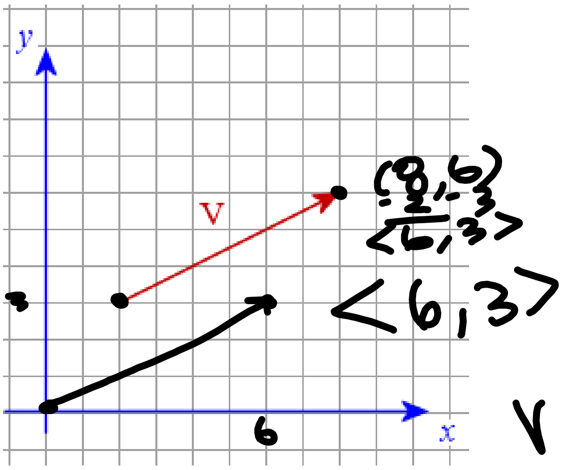




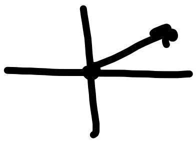
Component Form

From origin (Initial Point)
Terminal Point written as <x,y>

left 2
down 3



v <6,3>



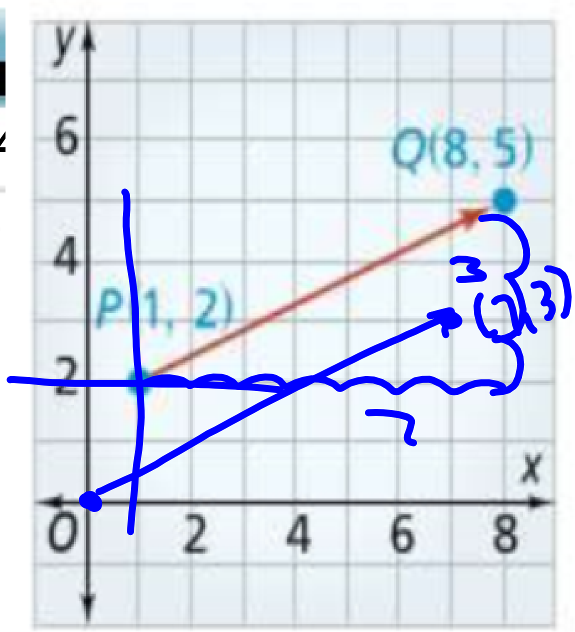
Launch — Instruction — Practice — Self Assessment

• Problem 1 • 2 • 3 • 4 • 5

Problem 1 **Representing a Vector** pg H4

What is the component form of the vector v = $\overrightarrow{PQ}$ shown here?

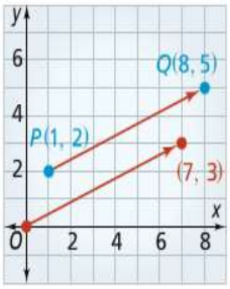
<7, 3>



PH48

Problem 1 Representing a Vector

What is the component form of the vector $v = \overrightarrow{PQ}$ shown here?



Think

For component form, the initial point must be at the origin. You need to move it 1 unit left and 2 units down.

Write: $(1 - 1, 2 - 2) = (0, 0)$

Think

You must move the terminal point in the same way.

Write: $(8 - 1, 5 - 2) = (7, 3)$

Think

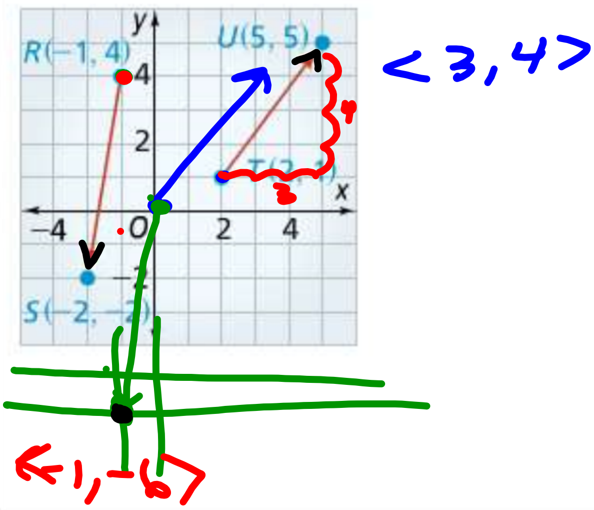
The new terminal point is the component form.

Write: $\mathbf{v} = \langle 7, 3 \rangle$

PH48

Got It? Got it pg H48

What are the component forms of the two vectors shown here?

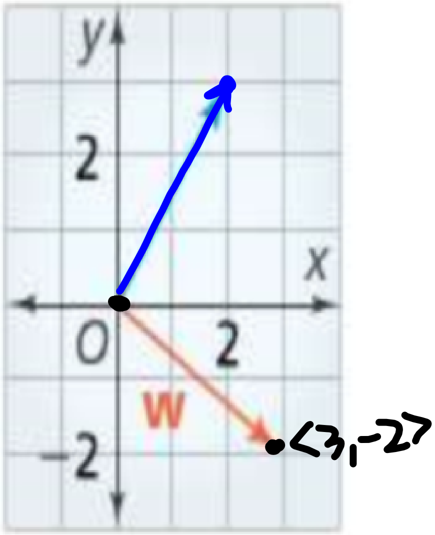


PH48

Problem 2 Rotating a Vector pg H48

Rotate the vector $\boxed{w = \langle 3, -2 \rangle}$ by 90°. What is the component form of the resulting vector?

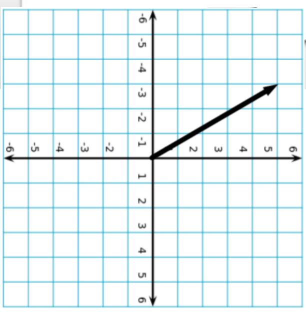
<2,3>

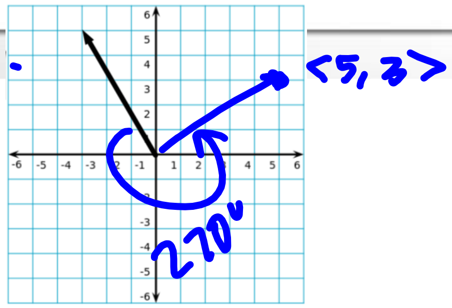


PH48

Got It? got it pg H49

Rotate the vector v = ⟨−3, 5⟩ by 270°. What is the component form of the resulting vector?





b. Reasoning What other matrix transformations can you apply to vectors in matrix form?

PH49

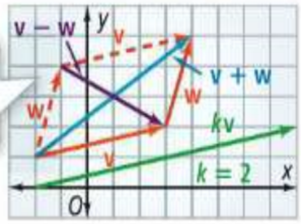
You can use real number operations to define operations involving vectors. pg H49

take note **Properties Operations With Vectors**

Given $\mathbf{v} = \langle v_1, v_2 \rangle$, $\mathbf{w} = \langle w_1, w_2 \rangle$, and any real number k:

$\mathbf{v} + \mathbf{w} = \langle v_1 + w_1, v_2 + w_2 \rangle$
$\mathbf{v} - \mathbf{w} = \langle v_1 - w_1, v_2 - w_2 \rangle$
$k\mathbf{v} = \langle kv_1, kv_2 \rangle$

Note that $\mathbf{w} + (\mathbf{v} - \mathbf{w}) = \mathbf{v}$ and $(\mathbf{v} - \mathbf{w}) + \mathbf{w} = \mathbf{v}$.



To find magnitude: Put in component form and use Pythagorean Theorem to find length.

The position of a vector is not important. For this reason, a vector **v** in standard position has initial point (0, 0) and is completely determined by its terminal point (a, b). You can represent **v** in component form as $\langle a, b \rangle$. Use the Pythagorean theorem to find the magnitude of **v**, $|\mathbf{v}| = \sqrt{a^2 + b^2}$.

PH49

pg H49

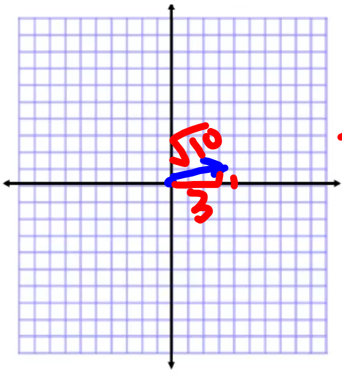
Problem 3 Adding and Subtracting Vectors

GRIDDED RESPONSE Let $\underline{u} = \langle -2, 3 \rangle$ and $\underline{v} = \langle 5, -2 \rangle$. What is $|\underline{u} + \underline{v}|$, rounded to the nearest hundredth?

Video

$\langle -2+5, 3+(-2) \rangle$

$|\langle 3, 1 \rangle|$



$3^2 + 1^2 = c^2$

$9 + 1 = c^2$

$\sqrt{10} = \sqrt{c^2}$

PH49

pg H49

Problem 3 Adding and Subtracting Vectors

GRIDDED RESPONSE Let $u = \langle -2, 3 \rangle$ and $v = \langle 5, -2 \rangle$. What is $|u + v|$, rounded to the nearest hundredth?

To find $\mathbf{u} + \mathbf{v}$, use the tip-to-tail method.

Given $\mathbf{v} = \langle v_1, v_2 \rangle$ and $\mathbf{w} = \langle w_1, w_2 \rangle$, the vector sum $\mathbf{v} + \mathbf{w} = \langle v_1 + w_1, v_2 + w_2 \rangle$.
Graphically, this is the same as placing the tail of **w** at the tip of **v**.
The resultant vector **v** + **w**, drawn from the tail of **v** to the tip of **w**, has the component form $\langle v_1 + w_1, v_2 + w_2 \rangle$.

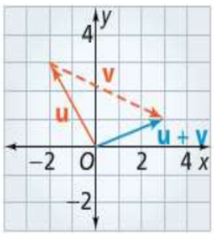
PH49

pg H49

Problem 3 Adding and Subtracting Vectors

GRIDDED RESPONSE Let $\mathbf{u} = \langle -2, 3 \rangle$ and $\mathbf{v} = \langle 5, -2 \rangle$. What is $|\mathbf{u} + \mathbf{v}|$, rounded to the nearest hundredth?

To find $\mathbf{u} + \mathbf{v}$, use the tip-to-tail method.



Step 1 Draw $\mathbf{u} = \langle -2, 3 \rangle$ in standard position.

Step 2 At the tip of $\mathbf{u}$, draw $\mathbf{v} = \langle 5, -2 \rangle$ from (−2, 3) to (3, 1).

Step 3 Draw $\mathbf{u} + \mathbf{v}$ to have the initial point of $\mathbf{u}$ and the terminal point of $\mathbf{v}$.

Step 4 Express $\mathbf{u} + \mathbf{v}$ in component form. $\mathbf{u} + \mathbf{v} = \langle 3, 1 \rangle$

Step 5 Find $|\mathbf{u} + \mathbf{v}|$. $|\mathbf{u} + \mathbf{v}| = \sqrt{3^2 + 1^2} = \sqrt{10} \approx 3.16$

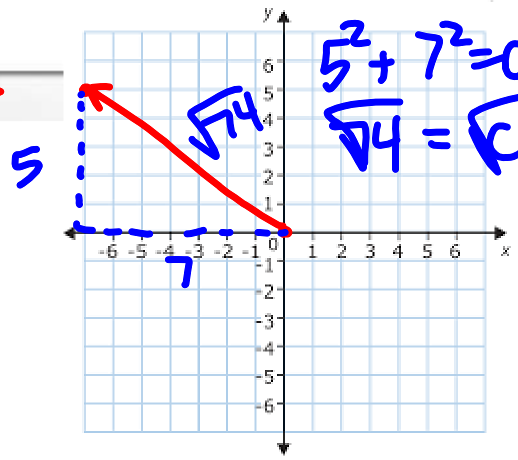
PH49

Got It? pg H49

Let u = $\langle -2, 3\rangle$ and v = $\langle 5, -2\rangle$. What is $|u - v|$, rounded to the nearest hundredth?

$u - v \quad \langle -2-5, 3-(-2)\rangle$

$|\langle -7, 5\rangle|$



$5^2 + 7^2 = c^2$

$\sqrt{74} = \sqrt{c^2}$

PH49

Scalar multiplication of a vector by a positive number (other than 1) changes only the magnitude. Multiplication by a negative number (other than -1) changes the magnitude and reverses the direction of the vector.

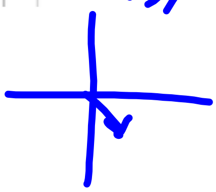
pg H49-50

Problem 4 **Scalar Multiplication**

For v $3\langle 1, -2\rangle$ and w = $\langle 2, 3\rangle$, what is the graph of the following vectors?

A v and 3v

$\langle 3, -6\rangle$ Shown on next slide...



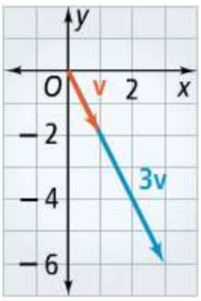
PH49-50

Problem 4 Scalar Multiplication pg H50

For $\mathbf{v} = \langle 1, -2 \rangle$ and $\mathbf{w} = \langle 2, 3 \rangle$, what is the graph of the following vectors?

A v and 3v

$$3\mathbf{v} = 3\langle 1, -2 \rangle$$
$$= \langle 3(1), 3(-2) \rangle$$
$$= \langle 3, -6 \rangle$$



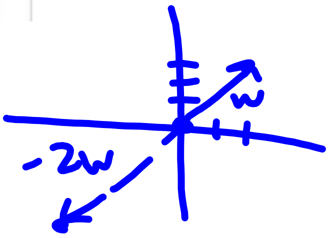
PH50

Problem 4 Scalar Multiplication pg H50

For $v = \langle 1, -2 \rangle$ and $w = -2\langle 2, 3 \rangle$, what is the graph of the following vectors?

B w and -2w $\langle -4, -6 \rangle$

Shown on next slide



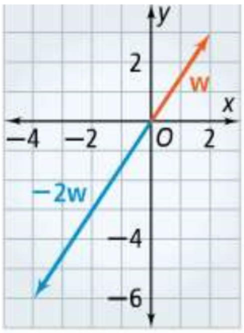
PH50

Problem 4 **Scalar Multiplication** pg H50

For $\mathbf{v} = \langle 1, -2 \rangle$ and $\mathbf{w} = \langle 2, 3 \rangle$, what is the graph of the following vectors?

B $\mathbf{w}$ and $-2\mathbf{w}$

$$\begin{aligned} -2\mathbf{w} &= -2\langle 2, 3 \rangle \\ &= \langle -2(2), -2(3) \rangle \\ &= \langle -4, -6 \rangle \end{aligned}$$



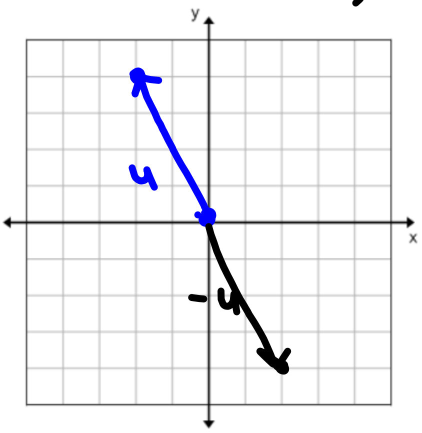
PH50

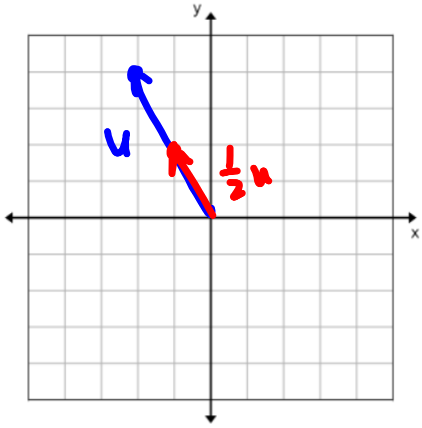
Got it pg H50

Got It? **4.** Given $\mathbf{u} = \frac{1}{2} \cdot \langle -2, 4 \rangle$, what are the graphs of the following vectors?

a. $-\mathbf{u}$ $\langle 2, -4 \rangle$

b. $\frac{1}{2}\mathbf{u}$ $\langle -1, 2 \rangle$





PH50

If $\mathbf{v} = \langle v_1, v_2 \rangle$ and $\mathbf{w} = \langle w_1, w_2 \rangle$, the **dot product** $\mathbf{v} \cdot \mathbf{w}$ is $v_1w_1 + v_2w_2$. If $\mathbf{v} \cdot \mathbf{w} = 0$, the two vectors are **normal**, or perpendicular, to each other.

Problem 5 **Finding Dot Products** pg H50

Are the following vectors normal?

$-\frac{5}{2}$ $\frac{3}{7}$

A $t = \langle 2, -5 \rangle$, $u = \langle 7, 3 \rangle$

$2 \cdot 7 + -5 \cdot 3$

$14 + (-15)$

$= -1$

$-\frac{4}{10} = -\frac{2}{5}$ $\frac{5}{2}$

B $v = \langle 10, -4 \rangle$, $w = \langle 2, 5 \rangle$

$10 \cdot 2 + -20$

$20 + (-20)$

$= 0$

PH50

Got it pg H50

Got It? **5.** Are the following vectors normal?

a. $\langle -2, 6\rangle, \langle -9, -18\rangle$

-90

b. $\left\langle 3, \frac{5}{6}\right\rangle, \left\langle -\frac{10}{9}, 4\right\rangle$

$\frac{\cancel{3}^{1}}{1}\left(-\frac{10}{\cancel{9}_{3}}\right) + \frac{5}{\cancel{6}_{3}}\left(\frac{\cancel{4}^{2}}{1}\right)$

$-\frac{10}{3} + \frac{10}{3}$

$= 0$

PH50

hw A6 #s 1-29 odds