Bell Ringer

Section A-3

Determine whether the matrices are multiplicative inverses.

1. $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ *and* $\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$

$A \cdot A^{-1} = I$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Find the inverse matrix if it exists.

2. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

$2-3=-1$

$\frac{1}{-1}\begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

3. $\begin{bmatrix} 6 & -8 \\ -3 & 4 \end{bmatrix}$

$\det = 24-24=0$

No inverse!

Review.

4. Solve the system. $2(3x + y = 5)$
$2x - \cancel{2y} = -2$

$+ \quad 6x + \cancel{2y} = 10$

$\frac{8x}{8} = \frac{8}{8}$

$x = 1$

$\cancel{3} + y = 5$
$-\cancel{3} \quad -3$

$y = 2$

$(1, 2)$

Solutions

Section A-3

Determine whether the matrices are multiplicative inverses.

1. $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ and $\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$ Yes

Find the inverse matrix if it exists.

2. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

3. $\begin{bmatrix} 6 & -8 \\ -3 & 4 \end{bmatrix}$ Does not have an inverse

Review.

4. Solve the system. $\begin{aligned} 3x + y &= 5 \\ 2x - 2y &= -2 \end{aligned}$ (1,2)

Recap...

Write the identity matrix for a 2x2

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find detA

$$A = \begin{bmatrix} 1 & 2 \\ -4 & 4 \end{bmatrix}$$

$1 \cdot 4 - 2(-4)$

$4 + 8 = 12$

Find A^{-1}

$$A = \begin{bmatrix} 1 & 2 \\ -4 & 4 \end{bmatrix}$$

det=12

$$A^{-1} = \frac{1}{12}\begin{bmatrix} 4 & -2 \\ 4 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{6} \\ \frac{1}{3} & \frac{1}{12} \end{bmatrix}$$

If A and A^{-1} are inverse matrices, what is $A \bullet A^{-1}$?

Identity

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix} \qquad 1\cdot 0 - 2\cdot 3 = 0 - 6 = -6$$

$$\frac{1}{-6}\begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & -\frac{1}{6} \end{bmatrix}}_{A^{-1}}$$

Can you find:

A + B   C - A   B - C   AB   CB   BA

$$A = \begin{bmatrix} -2 & -6 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 5 & -8 & 1 \\ 5 & 2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 6 & -4 \\ -9 & 7 \end{bmatrix}$$

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A-4 Inverse Matrices and Systems

Content Standard

N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

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Normally: Solve $\frac{\cancel{A}X}{\cancel{A}} = \frac{B}{A}$ for X

$X = \frac{B}{A}$

With Matrices: Solve $\cancel{A^{-1}A}X = A^{-1}B$ for X

$X = A^{-1}B$

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Problem 1 Solving Equations Using an Inverse Matrix

What is the solution of the matrix equation?

$AX = B$

A $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ $\quad X = \begin{bmatrix} 11 \\ -18 \end{bmatrix}$

$10-9=1 \quad \frac{1}{1}\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} = A^{-1}$

Think
How do you know the equation has a solution?

Step 1 Evaluate det A and find A^{-1}.

$$\det A = ad - bc = (5)(2) - (3)(3) = 1$$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Think
Does it matter if you multiply A^{-1} on the left or right side of A?

Step 2 Multiply each side of the equation by A^{-1}.

$$\underset{A^{-1}}{\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}}\underset{A}{\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}} X = \underset{A^{-1}}{\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}}\underset{B}{\begin{bmatrix} 1 \\ -3 \end{bmatrix}}$$

$$\begin{bmatrix} 2+9 \\ -3+-15 \end{bmatrix} = \begin{bmatrix} 11 \\ -18 \end{bmatrix}$$

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Problem 1 Solving Equations Using an Inverse Matrix

What is the solution of the matrix equation?

B $\begin{bmatrix} 3 & -9 \\ -2 & 6 \end{bmatrix} X = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

Evaluate det A.

$\det A = ad - bc = (3)(6) - (-9)(-2) = 0$

Matrix A has no inverse. The equation has no solution.

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Got it pg H32

1. What is the solution of each matrix equation?

a. $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} X = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$ **b.** $\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} X = \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix}$ **c.** $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} X = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

det = 2

$8 - 6$

$\frac{1}{2}\begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & -\frac{3}{2} \\ -1 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & -\frac{3}{2} \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} -5 + -3 \\ 5 + 4 \end{bmatrix} = \begin{bmatrix} -8 \\ 9 \end{bmatrix} = X$

Got it pg H32

1. What is the solution of each matrix equation?

a. $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} X = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$ **b.** $\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} X = \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix}$ **c.** $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} X = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

A^{-1} A^{-1}

$21 - 20 = 1 = \text{det}$

Inverse: $\frac{1}{\text{det}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$

$\left(\frac{1}{3}\right) 3x = 4 \left(\frac{1}{3}\right)$

$A^{-1} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -14 \\ 19 \end{bmatrix}$

$\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 3(-3) + (-5)(1) & 0(3) + 4(-5) \\ -3(-4) + 7(1) & 0(-4) + 4(7) \end{bmatrix}$

$\begin{bmatrix} -14 \\ 19 \end{bmatrix}$

You can write a system of equations as a matrix equation $AX = B$, using a **coefficient matrix**, a **variable matrix**, and a **constant matrix**.

System of Equations

$$\begin{cases} 2x + 3y = 1 \\ 5x - 2y = 13 \end{cases}$$

Matrix Equation

$$A^{-1}\underbrace{\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}}_{\text{coefficient matrix, } A}\underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{variable matrix, } X} = A^{-1}\underbrace{\begin{bmatrix} 1 \\ 13 \end{bmatrix}}_{\text{constant matrix, } B}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix}$$

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Problem 2 Writing Systems as a Matrix Equation

What is the matrix equation that corresponds to the system?

A $\begin{cases} 4x + 7y = 6 \\ -5x + 3y = 1 \end{cases}$

Step 1 Identify the coefficient, variable, and constant matrices.

coefficient matrix, A	variable matrix, X	constant matrix, B
$\begin{bmatrix} 4 & 7 \\ -5 & 3 \end{bmatrix}$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$\begin{bmatrix} 6 \\ 1 \end{bmatrix}$

Step 2 Write the matrix equation.

$$\begin{bmatrix} 4 & 7 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

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coefficient matrix, A variable matrix, X constant matrix, B

Problem 2 Writing Systems as a Matrix Equation

What is the matrix equation that corresponds to the system?

B $\begin{cases} 3a + 5b - 12c = 6 \\ 7b + 2c = 8 \\ 5a = 3c + 1 \end{cases}$

Step 1 Rewrite the system so the variables are in the same order in each equation.

$$\begin{cases} 3a + 5b - 12c = 6 \\ 7b + 2c = 8 \\ 5a = 3c + 1 \end{cases} \rightarrow \begin{cases} 3a + 5b - 12c = 6 \\ 7b + 2c = 8 \\ 5a - 3c = 1 \end{cases}$$

Step 2 Identify the coefficient, variable, and constant matrices.

coefficient matrix, A variable matrix, X constant matrix, B

$$\begin{bmatrix} 3 & 5 & -12 \\ 0 & 7 & 2 \\ 5 & 0 & -3 \end{bmatrix} \qquad \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad \begin{bmatrix} 6 \\ 8 \\ 1 \end{bmatrix}$$

Step 3 Write the matrix equation.

$$\begin{bmatrix} 3 & 5 & -12 \\ 0 & 7 & 2 \\ 5 & 0 & -3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 1 \end{bmatrix}$$

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Got it pg H33

2. What is the matrix equation that corresponds to each system?

a. $\begin{cases} 3x - 7y = 8 \\ 5x + 1y = -2 \end{cases}$ **b.** $\begin{cases} x + 3y + 5z = 12 \\ -2x + y - 4z = -2 \\ 7x - 2y = 7 \end{cases}$ **c.** $\begin{cases} 2x + 3 = 8y \\ -x + y = -4 \end{cases}$

$-8y \quad -3$

$A^{-1}\begin{bmatrix} 3 & -7 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\begin{bmatrix} 8 \\ -2 \end{bmatrix}$

I

det $= 3 - -35 = 38$

$A^{-1} = \frac{1}{38}\begin{bmatrix} 1 & 7 \\ -5 & 3 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{38} & \frac{7}{38} \\ \frac{-5}{38} & \frac{3}{38} \end{bmatrix}\begin{bmatrix} 8 \\ -2 \end{bmatrix} = \begin{bmatrix} -\frac{3}{19} \\ -\frac{23}{19} \end{bmatrix}$

$\frac{8}{38} + \frac{-14}{38} = -\frac{6}{38} = -\frac{3}{19}$

$-\frac{40}{38} + \frac{-6}{38} = -\frac{46}{38} = -\frac{23}{19}$

$\left(-\frac{3}{19}, \frac{-23}{19}\right)$

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Problem 3 Solving a System of Two Equations

What is the solution of the system $\begin{cases} 5x - 4y = 4 \\ 3x - 2y = 3 \end{cases}$? Solve using matrices.

Got it pg H34

3. What is the solution of each system of equations? Solve using matrices.

a. $\begin{cases} 9x + 2y = 3 \\ 3x + y = -6 \end{cases}$ **b.** $\begin{cases} 4x - 6y = 9 \\ -10x + 15y = 8 \end{cases}$

$$A^{-1}\begin{bmatrix} 9 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\begin{bmatrix} 3 \\ -6 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

$9-6=3$

$$\frac{1}{3}\begin{bmatrix} 1 & -2 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -6 \end{bmatrix} = \begin{bmatrix} 1+4 \\ -3-18 \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

What if you set up a matrix to solve a system of equations and the determinant is 0??!

$$\begin{bmatrix} -6 & 3 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \end{bmatrix}$$

hw A4 - #s 1-21 odds, skip #9, 46