Bell Ringer - Just do #1, we haven't learned the rest :)

Section A-3

Determine whether the matrices are multiplicative inverses.

1. $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ *and* $\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A (2x2) A^{-1} (2x2)

Yes!

Does AB = **I**? [

Find the inverse matrix if it exists.

2. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 6 & -8 \\ -3 & 4 \end{bmatrix}$

Review.

4. Solve the system.

$$3x + y = 5$$
$$2x - 2y = -2$$

Solutions

Section A-3

Determine whether the matrices are multiplicative inverses.

1. $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ *and* $\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$ Yes

Find the inverse matrix if it exists.

2. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

3. $\begin{bmatrix} 6 & -8 \\ -3 & 4 \end{bmatrix}$ Does not have an inverse

Review.

4. Solve the system. $3x + y = 5$, $2x - 2y = -2$ (1,2)

Identity Matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Determinants...

The **determinant** of a 2 × 2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is det $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ = $ad - bc$.

pg H24

A What is det $\begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$?

$ad - bc$

$\det = 3 \cdot 5 - (-1)(2)$

$15 + 2$

$= 17$

pg H24

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ det = ad − bc

Got It? **2.** What are the determinants of the following matrices?

a. $\begin{bmatrix} 3 & 6 \\ 2 & 5 \end{bmatrix} = 3$ **b.** $\begin{bmatrix} -2 & 0 \\ 3 & 0 \end{bmatrix}$ **c.** $\begin{bmatrix} 1 & [illegible] & 3 \\ 2 & 4 & 6 \\ [illegible] & -1 & [illegible] \end{bmatrix}$ X

$3 \cdot 5 - 6 \cdot 2$

$15 - 12$

$= 3$

$-2 \cdot 0 - 0 \cdot 3$

$0 - 0$

$= 0$

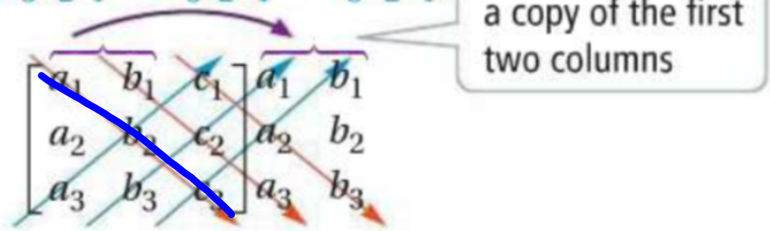
H 24

Determinants...

The determinant of a 3×3 matrix $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is

$$a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - (a_3b_2c_1 + b_3c_2a_1 + c_3a_2b_1)$$

Visualize the pattern this way:



pg H24

B What is det $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 4 & -1 \\ 3 & 5 & 2 \end{bmatrix}$?

$\begin{matrix} 1 & 0 \\ 0 & 4 \\ 3 & 5 \end{matrix}$

det = 8 + 0 + 0 − (−24 + −5 + 0)

= 8 − (−29)

= 37

Find the determinant pg H24

Calculator...

c. $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 6 \\ 5 & -1 & 3 \end{bmatrix}$ $= -48$

Inverse of a matrix: A^{-1}

$$AA^{-1} = A^{-1}A = \mathbf{I}$$

$AB^{-1} = B^{-1}A = I$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$2 \cdot 3 = 6$
$3 \cdot 2 = 6$

$AB \neq BA$

* Only square matrices have inverses

* Only square matrices are identity matrices

How to find an inverse matrix:

1 - Find the determinant

-if the determinant is 0, there is no inverse

-if the determinant is NOT 0, continue

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

det = $4 \cdot 1 - 3 \cdot 2$

$4 - 6 = -2$

How to find an inverse matrix:

2 - Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

det = -2

3 - $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Example

$A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{-2}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$

$\rightarrow A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{bmatrix}$

Inverse of a 2x2 Matrix...

The determinant of a matrix can help you determine whether the matrix has an inverse and, if it exists, to find the inverse.

take note

Key Concept Inverse of a 2 × 2 Matrix

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

If $\det A = 0$, then A is a **singular matrix** and has no inverse.

If $\det A \neq 0$, then the inverse of A, written A^{-1}, is

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

H 25

pg H26

Problem 4 Finding the Inverse of a Matrix

Does the matrix $A = \begin{bmatrix} -3 & 6 \\ -1 & 3 \end{bmatrix}$ have an inverse? If it does, what is A^{-1}?

Inverse of matrix

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

det

$\det A = -9 + 6 = -3$

$A^{-1} = -\frac{1}{3}\begin{bmatrix} 3 & -6 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -\frac{1}{3} & 1 \end{bmatrix} = A^{-1}$

$AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

pg H26

Inverse of matrix

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

det A

Got It? **4.** Does the matrix have an inverse? If so, what is it?

a. $A = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 \\ -1.5 & 2 \end{bmatrix}$

b. $B = \begin{bmatrix} 2 & 5 \\ -4 & -10 \end{bmatrix}$ no inverse

c. $C = \begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}$

$8-6=2$ $\begin{bmatrix} 6 & 9 \end{bmatrix}$

det $=0$

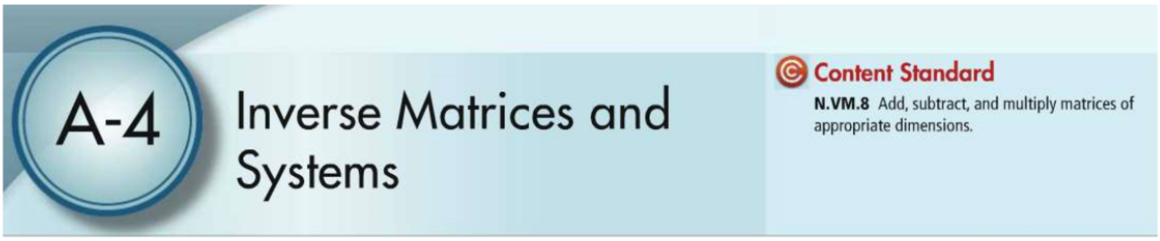
$-20 - -20$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & -1 \\ -1.5 & 2 \end{bmatrix}$

hw A3 # 1-19 odds, 29-35 odds

pg H31


A-4
Inverse Matrices and Systems
Content Standard
N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

PH31

In this lesson, you will solve systems of equations by solving a matrix equation.

Essential Understanding You can solve some matrix equations $AX = B$ by multiplying each side of the equation by A^{-1}, the inverse of matrix A.

If matrix A has an inverse, you can use it to solve the matrix equation $AX = B$. Multiply each side of the equation by A^{-1} to find X.

$AX = B$

$A^{-1}AX = A^{-1}B$ Multiply each side by A^{-1}.

$IX = A^{-1}B$ $A^{-1}A = I$, the identity matrix.

$X = A^{-1}B$ $IX = X$

PH31

Problem 1 Solving Equations Using an Inverse Matrix

What is the solution of the matrix equation?

A $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

Think
How do you know the equation has a solution?

Step 1 Evaluate det A and find A^{-1}.

$$\det A = ad - bc = (5)(2) - (3)(3) = 1$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Think
Does it matter if you multiply A^{-1} on the left or right side of A?

Step 2 Multiply each side of the equation by A^{-1}.

$$\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

PH31

Problem 1 **Solving Equations Using an Inverse Matrix**

What is the solution of the matrix equation?

B $\begin{bmatrix} 3 & -9 \\ -2 & 6 \end{bmatrix} X = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

Evaluate det A.

$\det A = ad - bc = (3)(6) - (-9)(-2) = 0$

Matrix A has no inverse. The equation has no solution.

PH32

Got It? **1.** What is the solution of each matrix equation?

a. $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} X = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$ **b.** $\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} X = \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix}$ **c.** $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} X = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

PH32

You can write a system of equations as a matrix equation $AX = B$, using a **coefficient matrix**, a **variable matrix**, and a **constant matrix**.

System of Equations

$$\begin{cases} 2x + 3y = 1 \\ 5x - 2y = 13 \end{cases}$$

Matrix Equation

$$\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 13 \end{bmatrix}$$

coefficient matrix, *A* | variable matrix, *X* | constant matrix, *B*

PH32

Problem 2 Writing Systems as a Matrix Equation

What is the matrix equation that corresponds to the system?

A $\begin{cases} 4x + 7y = 6 \\ -5x + 3y = 1 \end{cases}$

Step 1 Identify the coefficient, variable, and constant matrices.

coefficient matrix, A: $\begin{bmatrix} 4 & 7 \\ -5 & 3 \end{bmatrix}$

variable matrix, X: $\begin{bmatrix} x \\ y \end{bmatrix}$

constant matrix, B: $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$

Step 2 Write the matrix equation.

$$\begin{bmatrix} 4 & 7 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

PH33

coefficient matrix, A variable matrix, X constant matrix, B

Problem 2 Writing Systems as a Matrix Equation

What is the matrix equation that corresponds to the system?

B $\begin{cases} 3a + 5b - 12c = 6 \\ 7b + 2c = 8 \\ 5a = 3c + 1 \end{cases}$

Step 1 Rewrite the system so the variables are in the same order in each equation.

$$\begin{cases} 3a + 5b - 12c = 6 \\ 7b + 2c = 8 \\ 5a = 3c + 1 \end{cases} \rightarrow \begin{cases} 3a + 5b - 12c = 6 \\ 7b + 2c = 8 \\ 5a - 3c = 1 \end{cases}$$

Step 2 Identify the coefficient, variable, and constant matrices.

coefficient matrix, A: $\begin{bmatrix} 3 & 5 & -12 \\ 0 & 7 & 2 \\ 5 & 0 & -3 \end{bmatrix}$

variable matrix, X: $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

constant matrix, B: $\begin{bmatrix} 6 \\ 8 \\ 1 \end{bmatrix}$

Step 3 Write the matrix equation.

$$\begin{bmatrix} 3 & 5 & -12 \\ 0 & 7 & 2 \\ 5 & 0 & -3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 1 \end{bmatrix}$$

PH33

Got It? **2.** What is the matrix equation that corresponds to each system?

a. $\begin{cases} 3x - 7y = 8 \\ 5x + y = -2 \end{cases}$

b. $\begin{cases} x + 3y + 5z = 12 \\ -2x + y - 4z = -2 \\ 7x - 2y = 7 \end{cases}$

c. $\begin{cases} 2x + 3 = 8y \\ -x + y = -4 \end{cases}$

PH33

Problem 3 Solving a System of Two Equations

What is the solution of the system $\begin{cases} 5x - 4y = 4 \\ 3x - 2y = 3 \end{cases}$? Solve using matrices.

Think

Write

Write the system as a matrix equation. Write the coefficient, variable, and constant matrices.

You need to find A^{-1}.

Multiply each side of the equation by A^{-1} on the left.

Solve for $\begin{bmatrix} x \\ y \end{bmatrix}$ and check.

PH34

Got It? **3.** What is the solution of each system of equations? Solve using matrices.

a. $\begin{cases} 9x + 2y = 3 \\ 3x + y = -6 \end{cases}$

b. $\begin{cases} 4x - 6y = 9 \\ -10x + 15y = 8 \end{cases}$

PH34

The system $\begin{cases} -6x + 3y = 8 \\ 4x - 2y = 10 \end{cases}$ has coefficient matrix A with $\det A = 0$. There is no inverse matrix and the system has no unique solution. Recall that this means the system either has no solutions (graphs are parallel lines in the 2×2 case) or infinitely many solutions (graphs are the same line in the 2×2 case). For the system above, the lines are parallel.

PH34

Problem 4 Solving a System of Three Equations

Multiple Choice On a new exercise program, your friend plans to do a run-jog-walk routine every other day for 40 min. She would like to burn 310 calories during each session. The table shows how many calories a person your friend's age and weight burns per minute of each type of exercise.

Calories Burned

Running (8 mi/h)	Jogging (5 mi/h)	Walking (3.5 mi/h)
12.5 cal/min	7.5 cal/min	3.5 cal/min

If your friend plans on jogging twice as long as she runs, how many minutes should she exercise at each rate?

(A) run 10, jog 5, walk 25

(B) run 30, jog 15, walk 5

(C) run 5, jog 10, walk 25

(D) run 10, jog 20, walk 10

PH35

Problem 4 Solving a System of Three Equations

Multiple Choice On a new exercise program, your friend plans to do a run-jog-walk routine every other day for 40 min. She would like to burn 310 calories during each session. The table shows how many calories a person your friend's age and weight burns per minute of each type of exercise.

Step 1 Define the variables. Let x = number of minutes running.
y = number of minutes jogging.
z = number of minutes walking.

Step 2 Write a system of equations for the problem.

$$\begin{cases} 12.5x + 7.5y + 3.5z = 310 \\ x + y + z = 40 \\ 2x = y \end{cases} \rightarrow \begin{cases} 12.5x + 7.5y + 3.5z = 310 \\ x + y + z = 40 \\ 2x - y + 0z = 0 \end{cases}$$

Step 3 Write the system as a matrix equation.

$$\begin{bmatrix} 12.5 & 7.5 & 3.5 \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 310 \\ 40 \\ 0 \end{bmatrix}$$

Step 4 Use a calculator. Solve for the variable matrix.

Step 5 Interpret the solution.

PH35

Got It? **4.** After following her exercise program from Problem 4 for a month, your friend plans to increase the calories she burns with each session. She still wants to exercise for 40 min every other day, but now she wants to burn 460 calories during each session. If she only runs and jogs, how many minutes of each exercise type should she do now?

PH36

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Solve each matrix equation. If an equation cannot be solved, explain why.

See Problem 1.

7. $\begin{bmatrix} 12 & 7 \\ 5 & 3 \end{bmatrix} X = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

8. $\begin{bmatrix} 0 & -4 \\ 0 & -1 \end{bmatrix} X = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

9. $\begin{bmatrix} 5 & 1 & 4 \\ 2 & -3 & -5 \\ 7 & 2 & -6 \end{bmatrix} X = \begin{bmatrix} 5 \\ 2 \\ 5 \end{bmatrix}$

10. $\begin{bmatrix} 6 & 10 & 13 \\ 4 & -2 & 7 \\ 0 & 9 & -8 \end{bmatrix} X = \begin{bmatrix} 84 \\ 18 \\ 56 \end{bmatrix}$

Write each system as a matrix equation. Identify the coefficient matrix, the variable matrix, and the constant matrix.

See Problem 2.

11. $\begin{cases} x + y = 5 \\ x - 2y = -4 \end{cases}$

12. $\begin{cases} y = 3x - 7 \\ x = 2 \end{cases}$

13. $\begin{cases} 3a + 5b = 0 \\ a + b = 2 \end{cases}$

14. $\begin{cases} x + 3y - z = 2 \\ x + 2z = 8 \\ 2y - z = 1 \end{cases}$

15. $\begin{cases} r - s + t = 150 \\ 2r + t = 425 \\ s + 3t = 0 \end{cases}$

16. $\begin{cases} x + 2y = 11 \\ 2x + 3y = 18 \end{cases}$

PH36

Solve each system of equations using a matrix equation. Check your answers. See Problem 3.

17. $\begin{cases} x + 3y = 5 \\ x + 4y = 6 \end{cases}$

18. $\begin{cases} p - 3q = -1 \\ -5p + 16q = 5 \end{cases}$

19. $\begin{cases} 300x - y = 130 \\ 200x + y = 120 \end{cases}$

20. $\begin{cases} x + 5y = -4 \\ x + 6y = -5 \end{cases}$

21. $\begin{cases} 2x + 3y = 12 \\ x + 2y = 7 \end{cases}$

22. $\begin{cases} 2x + 3y = 5 \\ x + 2y = 6 \end{cases}$

23. $\begin{cases} x + y + z = 4 \\ 4x + 5y = 3 \\ y - 3z = -10 \end{cases}$

24. $\begin{cases} 9y + 2z = 18 \\ 3x + 2y + z = 5 \\ x - y = -1 \end{cases}$

25. $\begin{cases} 9y + 2z = 14 \\ 3x + 2y + z = 5 \\ x - y = -1 \end{cases}$

26. Fitness Your classmate is starting a new fitness program. He is planning to ride his bicycle 60 minutes every day. He burns 7 Calories per minute bicycling at 11 mph and 11.75 Calories per minute bicycling at 15 mph. How long should he bicycle at each speed to burn 600 calories per hour? See Problem 4.

27. Think About a Plan Suppose you want to fill nine 1-lb tins with a snack mix. You plan to buy almonds for \$2.45/lb, peanuts for \$1.85/lb, and raisins for \$.80/lb. You want the mix to contain twice as much nuts as raisins by weight. If you spend exactly \$15, how much of each ingredient should you buy?

- How many equations do you need to represent this situation?
- How can you represent this system using a matrix equation?

28. Nutrition Suppose you are making a trail mix for your friends and want to fill three 1-lb bags. Almonds cost \$2.25/lb, peanuts cost \$1.30/lb, and raisins cost \$.90/lb. You want each bag to contain twice as much nuts as raisins by weight. If you spent \$4.45, how much of each ingredient did you buy?

PH37

hw A4 - #s 1-21 odds, skip #9, 45