No Bell Ringer

Pg H4 :)

A-1 Adding and Subtracting Matrices

Content Standards

N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 . . .

p. H4

A Matrix is an array of numbers

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

A Matrix

Label: Row x Column

$$\begin{bmatrix} -1 & 0 & 2 \\ 0 & 4 & 7 \\ -3 & -4 & -2 \end{bmatrix} \quad \begin{bmatrix} 6 & -2 & -5 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 7 \\ -3 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A B C D E

ELEMENTS... $a_{\text{row-column}}$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & & a_{mn} \end{pmatrix}$$

Label: Row x Column

$$\begin{bmatrix} -1 & 0 & 2 \\ 0 & 4 & 7 \\ -3 & -4 & -2 \end{bmatrix} \quad \begin{bmatrix} 6 & -2 & -5 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 7 \\ -3 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a_{33} =$ $\qquad a_{13} = -5 \qquad a_{11} = 5 \qquad a_{22} = 5 \qquad a_{21} = 0$

You can add and subtract matrices!!!!

take note

Key Concept **Matrix Addition and Subtraction**

To add matrices A and B with the same dimensions, add corresponding elements. Similarly, to subtract matrices A and B with the same dimensions, subtract corresponding elements.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix} \qquad A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{bmatrix}$$

p. H4

Problem 1 Adding and Subtracting Matrices

Given $C = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 4 & 3 \\ -2 & 2 & 4 \end{bmatrix}$, what are the following?

Think

To add matrices they need to have the same dimensions. What are the dimensions of C?

C has 2 rows and 3 columns, so it's a 2 × 3 matrix.

A $C + D$

$$\begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 3 \\ -2 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+1 & 2+4 & 4+3 \\ -1+(-2) & 4+2 & 0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & 7 \\ -3 & 6 & 4 \end{bmatrix}$$

p. H5

Problem 1 Adding and Subtracting Matrices

Given $C = \begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 4 & 3 \\ -2 & 2 & 4 \end{bmatrix}$, what are the following?

Think

To add matrices they need to have the same dimensions. What are the dimensions of C?

C has 2 rows and 3 columns, so it's a 2 × 3 matrix.

B $C - D$

$$\begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 4 & 3 \\ -2 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1 & 2-4 & 4-3 \\ -1-(-2) & 4-2 & 0-4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & -4 \end{bmatrix}$$

p. H5

Got It? **1.** Given $A = \begin{bmatrix} -12 & 24 \\ -3 & 5 \\ -1 & 10 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 2 & -4 \\ -1 & 5 \end{bmatrix}$, what are the following?

a. $A + B$

b. $A - B$

c. Reasoning Is matrix addition commutative? Explain.

A+B

a. $\begin{bmatrix} -15 & 25 \\ -1 & 1 \\ -2 & 15 \end{bmatrix}$

A-B

b. $\begin{bmatrix} -9 & 23 \\ -5 & 9 \\ 0 & 5 \end{bmatrix}$

p. H5

A **matrix equation** is an equation in which the variable is a matrix. You can use the addition and subtraction properties of equality to solve a matrix equation. An example of a matrix equation is shown below.

$$\begin{bmatrix} 1 & 0 & 12 \\ 3 & 5 & 9 \\ 7 & 8 & -2 \end{bmatrix} + A = \begin{bmatrix} 8 & 11 & 9 \\ -5 & 5 & 2 \\ 10 & 7 & 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 7 & 11 & -3 \\ -8 & 0 & -7 \\ 3 & -1 & 10 \end{bmatrix}$$

p. H5

Problem 2 Solving a Matrix Equation

Sports The first table shows the teams with the four best records halfway through their season. The second table shows the full season records for the same four teams. Which team had the best record during the second half of the season?

1st half + 2nd half = whole season

Records for the First Half of the Season

Team	Wins	Losses
Team 1	30	11
Team 2	29	12
Team 3	25	16
Team 4	24	17

Second Half

$$\begin{bmatrix} 23 & 18 \\ 38 & 3 \\ 33 & 8 \\ 37 & 4 \end{bmatrix}$$

Records for Season

Team	Wins	Losses
Team 1	53	29
Team 2	67	15
Team 3	58	24
Team 4	61	21

$$- \begin{bmatrix} 30 & 11 \\ 29 & 12 \\ 25 & 16 \\ 24 & 17 \end{bmatrix}$$

p. H5

pg H6

Got It? **2.** If $B = \begin{bmatrix} 1 & 6 & -1 \\ 2 & 6 & 1 \\ -1 & -2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 & 0 \\ -1 & -3 & 6 \\ 2 & 3 & -1 \end{bmatrix}$, and $A - B = C$, what is A?

$$A - [B] = [C] + [B]$$
$$+ [B]$$

$$A = \begin{bmatrix} 3 & 6 & -1 \\ 1 & 3 & 7 \\ 1 & 1 & 3 \end{bmatrix}$$

For $m \times n$ matrices, the additive identity matrix is the **zero matrix** O, or $O_{m \times n}$, with all elements zero. The *opposite*, or *additive inverse*, of an $m \times n$ matrix A is $-A$ where each element is the opposite of the corresponding element of A.

p. H6

Zero matrix (identity matrix)

All elements = 0

Opposite matrix (additive inverse)

All corresponding elements are exact opposites

Problem 3 Using Identity and Opposite Matrices

What are the following sums?

Think

How is this like adding real numbers?
Adding zero leaves the matrix unchanged. Adding opposites give you zero.

A $\begin{bmatrix} 1 & 2 \\ 5 & -7 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 2+0 \\ 5+0 & -7+0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5 & -7 \end{bmatrix}$

B $\begin{bmatrix} 2 & 8 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} -2 & -8 \\ 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2+(-2) & 8+(-8) \\ -3+3 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

p. H6

pg H6

Got It? **3.** What are the following sums?

a. $\begin{bmatrix} 14 & 5 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -14 & -5 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 10 & -5 \\ 0 & 2 & -3 \end{bmatrix}$

$\begin{bmatrix} -1 & 10 & -5 \\ 0 & 2 & -3 \end{bmatrix}$

p. H6

FYI - pg H7

Equal matrices have the same dimensions and equal corresponding elements. For example, $\begin{bmatrix} 0.25 & 1.5 \\ -3 & \frac{4}{5} \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{4} & 1\frac{1}{2} \\ -3 & 0.8 \end{bmatrix}$ are equal matrices. You can use the definition of equal matrices to find unknown values in matrix elements.

pg H7

Problem 4 Finding Unknown Matrix Values

Multiple Choice What values of x and y make the equation true?

$$\begin{bmatrix} 9 & 3x + 1 \\ 2y - 1 & 10 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ -5 & 10 \end{bmatrix}$$

$x = 5$, $y = -2$

$2y - 1 = -5$
$+1 \quad +1$
$\frac{2y}{2} = \frac{-4}{2}$
$y = -2$

$3x + 1 = 16$
$-1 \quad -1$
$\frac{3x}{3} = \frac{15}{3}$
$x = 5$

pg H7

Got It? **4.** What values of x, y, and z make the following equations true?

a. $\begin{bmatrix} x+3 & -2 \\ y-1 & x+1 \end{bmatrix} = \begin{bmatrix} 9 & -2 \\ 2y+5 & 7 \end{bmatrix}$

b. $\begin{bmatrix} z & -3 \\ 3x & 0 \end{bmatrix} - \begin{bmatrix} 10 & -4 \\ x & 2y+6 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 8 & 4y+12 \end{bmatrix}$

$y - 1 = 2y + 5$

$-y \; -5 \quad -y \; -5$

$-6 = y$

$z - 10 = 2$

$+10 \quad +10$

$3x - x = 8$

$\frac{2x}{2} = \frac{8}{2}$

p. H7

pg H12

A-2 Matrix Multiplication

Content Standards

N.VM.6 Use matrices to represent and manipulate data . . .

N.VM.7 Multiply matrices by scalars to produce new matrices . . .

Also N.VM.8, N.VM.9

p. H12

Scalar Multiplication

take note

Key Concept Scalar Multiplication

To multiply a matrix by a scalar c, multiply each element of the matrix by c.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \quad cA = \begin{bmatrix} ca_{11} & ca_{12} & ca_{13} \\ ca_{21} & ca_{22} & ca_{23} \end{bmatrix}$$

$$2 \cdot \begin{bmatrix} 10 & 6 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 2 \cdot 10 & 2 \cdot 6 \\ 2 \cdot 4 & 2 \cdot 3 \end{bmatrix} = \begin{bmatrix} 20 & 12 \\ 8 & 6 \end{bmatrix}$$

pg H13

Problem 1 **Using Scalar Products**

If $A = \begin{bmatrix} 2 & 8 & -3 \\ -1 & 5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 5 \\ 0 & 3 & -2 \end{bmatrix}$, what is $4A + 3B$?

$$4A + 3B = 4\begin{bmatrix} 2 & 8 & -3 \\ -1 & 5 & 2 \end{bmatrix} + 3\begin{bmatrix} -1 & 0 & 5 \\ 0 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 32 & -12 \\ -4 & 20 & 8 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 15 \\ 0 & 9 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 32 & 3 \\ -4 & 29 & 2 \end{bmatrix}$$

Got It? **1.** Using matrices A and B from Problem 1, what is $3A - 2B$?

p. H13

pg H13

Problem 2 Solving a Matrix Equation With Scalars

What is the solution of $2X + 3\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 11 & 0 \end{bmatrix}$?

$$2X + \begin{bmatrix} 6 & -3 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 11 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -3 \\ 9 & 12 \end{bmatrix}$$

$$\frac{1}{2} \cdot 2X = \begin{bmatrix} 2/2 & 8/2 \\ 2/2 & -12/2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 4 \\ 1 & -6 \end{bmatrix}$$

pg H13

Got It? **2.** What is the solution of $3X - 2\begin{bmatrix} -1 & 5 \\ 7 & 0 \end{bmatrix} = \begin{bmatrix} 17 & -13 \\ -7 & 0 \end{bmatrix}$?

$3X + \begin{bmatrix} 2 & -10 \\ -14 & 0 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$

$3x + 2 = 17$

$3x - 2(-1) = 17$

$3x - 2(5) = -13$

$3x - 10 = -13$

Matrix Multiplication

The product of two matrices is a matrix. To find an element in the product matrix, multiply the elements of a row from the first matrix by the corresponding elements of a column from the second matrix. Then add the products.

take note

Key Concept Matrix Multiplication

To find element c_{ij} of the product matrix AB, multiply each element in the ith row of A by the corresponding element in the jth column of B. Then add the products.

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

p. H14

Problem 3 Multiplying Matrices

If $A = \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix}$, what is AB?

Think
What relationship must exist between the numbers of elements in a row of *A* and a column of *B*?
They must be equal.

Step 1 Multiply the elements in the first row of A by the elements in the first column of B. Add the products and place the sum in the first row, first column of AB.

$$\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & \_ \\ \_ & \_ \end{bmatrix}$$

$2(-1) + 1(0) = -2$

Step 2 Multiply the elements in the first row of A by the elements in the second column of B. Add the products and place the sum in the first row, second column of AB.

$$\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ \_ & \_ \end{bmatrix}$$

$2(3) + 1(4) = 10$

Repeat Steps 1 and 2 with the second row of A to fill in row two of the product matrix.

p. H14

$$A = \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix}$$

$$AB \begin{bmatrix} 2(-1)+1(0) & 2(3)+1(4) \\ -3(-1)+(0)(0) & -3(3)+0(4) \end{bmatrix}$$

$$AB = \begin{bmatrix} -2 & 10 \\ 3 & -9 \end{bmatrix}$$

Problem 3 **Multiplying Matrices**

If $A = \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix}$, what is AB?

Step 3 $\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ 3 & \_ \end{bmatrix}$ $(-3)(-1) + 0(0) = 3$

Step 4 $\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ 3 & -9 \end{bmatrix}$ $(-3)(3) + 0(4) = -9$

The product of $\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}$ and $\begin{bmatrix} -1 & 3 \\ 0 & 4 \end{bmatrix}$ is $\begin{bmatrix} -2 & 10 \\ 3 & -9 \end{bmatrix}$.

p. H14

pg H14

Got It? **3.** If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}$, what are the following products?

a. AB **b.** BA

c. Reasoning Is matrix multiplication commutative? Explain.

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -6 & 0 \\ -9 & 11 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 7 \\ 6 & 8 \end{bmatrix}$$

p. H14

Problem 4 Applying Matrix Multiplication

Sports In 1966, Washington and New York (Giants) played the highest scoring game in National Football League history. The table summarizes the scoring. A touchdown (TD) is worth 6 points, a field goal (FG) is worth 3 points, a safety (S) is worth 2 points, and a point after touchdown (PAT) is worth 1 point. Using matrix multiplication, what was the final score?

	TD	FG	S	PAT
WASHINGTON	10	1	0	9
NEW YORK	6	0	0	5

Think
What is the meaning of each number in matrix *P*?
They are the point values for each type of score.

Know
- The number of each type of score
- The point value of each score

Need
The scoring summary and point values as matrices

Plan
Multiply the matrices to find each team's final score.

Step 1 Enter the information in matrices.

$$S = \begin{bmatrix} 10 & 1 & 0 & 9 \\ 6 & 0 & 0 & 5 \end{bmatrix} \qquad P = \begin{bmatrix} 6 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

Step 2 Use matrix multiplication. The final score is the product *SP*.

$$SP = \begin{bmatrix} 10 & 1 & 0 & 9 \\ 6 & 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10(6) + 1(3) + 0(2) + 9(1) \\ 6(6) + 0(3) + 0(2) + 5(1) \end{bmatrix} = \begin{bmatrix} 72 \\ 41 \end{bmatrix}$$

Step 3 Interpret the product matrix.

The first row of *SP* shows scoring for Washington, so the final score was Washington 72, New York 41.

p. H15

pg H15

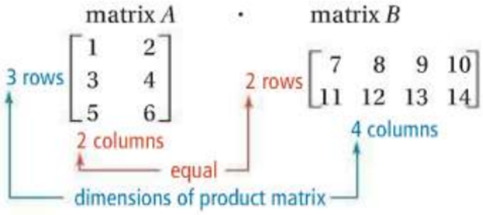
Got It? **4.** There are three ways to score in a basketball game: three-point field goals, two-point field goals, and one-point free throws. In 1994, suppose a high school player scored 36 two-point field goals and 28 free throws. In 2006, suppose a high school player scored 7 three-point field goals, 21 two-point field goals, and 18 free throws. Using matrix multiplication, how many points did each player score?

p. H15

take note

Property **Dimensions of a Product Matrix**

If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product matrix AB is an $m \times p$ matrix.



Product matrix AB is a 3×4 matrix.

p. H16

pg H16

Problem 5 Determining Whether Product Matrices Exist

Does either product AB or BA exist?

$$A = \begin{bmatrix} -2 & 1 \\ 3 & -2 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 & 2 & 1 \\ 2 & 0 & 0 & 3 \end{bmatrix}$$

(3x2) (2x4)

Think

How can you tell if a product matrix exists without computing it? Compare the dimensions of the matrices.

AB

$(3 \times 2)(2 \times 4) \rightarrow 3 \times 4$ product matrix

equal

Product AB exists.

BA

$(2 \times 4)(3 \times 2) \rightarrow$ no product

not equal

p. H16

pg H16

Got It? 5. Do the following products exist?

$A = \begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 4 & 2 & 0 \\ 1 & 3 & 5 \end{bmatrix}$

a. AB **b.** BA **c.** AC **d.** CA **e.** BC

p. H16

hw A1 - #s 2-24 evens, 28
hw A2 - #s 8-28 evens, 29-33 all