Bell Ringer

$y = a(x-h)^2 + k$

Wednesday 1/8

Write the function with the indicated transformations:

1. A quadratic function: Reflection across the x-axis, vertical shift up 3

$y = -x^2 + 3$

$y = a\sqrt{x-h} + k$

2. A square root function: vertical stretch by a factor of 3, horizontal shift left 2, vertical shift down 5

$y = 3\sqrt{x+2} - 5$

X

3. An absolute value function: reflection across the y-axis, horizontal shift right 5, vertical shift up 6

4. $(3+2i)+(5-6i)$

$5-6i$

$8-4i$

5. $(2-i)(6+2i)$

$12-2i-2(-1)$

$+2$

$14-2i$

	2	$-i$
6	12	$-6i$
$+2i$	$4i$	$-2i^2$

correct 5.2 ws

Name: Key

Sec. 5.2
Independent/Dependent Events

Determine if the following are dependent or independent events.

1. Hours you study for a test and your grade on that test
   dependent
2. Flipping a coin and rolling a die
   independent
3. Parking in a handicap parking space and getting a parking ticket
   dependent
4. Drawing an ace in a standard deck, put it back and then draw another
   independent
5. Jeremy took the SAT on Saturday and scored a 1350. The following week he took the ACT and scored a 23. independent
6. A card is randomly chosen from a deck of 52 cards, replaced, and a second card is chosen. independent
7. In a game, you roll an even number on a die and then you spin a spinner numbered 1 through 5 and get an odd number. independent
8. An ace is drawn, without replacement, from a deck of 52 cards. Then, a second ace is drawn. dependent

List event A and event B. Then determine if the following events are independent using the formula:

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

9. Use the sample space from number 14 to determine whether randomly getting a head on the coin and a purple marble are independent events.
   Event A: Head
   Event B: purple

   $P(\text{head and purple}) = P(\text{head}) \cdot P(\text{purple})$

   $\frac{2}{6} = \left(\frac{1}{2}\right) \cdot \left(\frac{4}{6}\right)$

   $\frac{1}{3} = \frac{1}{3}$

   Independent

10. Use the sample space from number 17 to determine whether randomly drawing two black marbles are independent events.
    Event A: Black on first draw
    Event B: Black on second draw

    $P(\text{Black and Black}) = P(\text{Black on first}) \cdot P(\text{Black on second})$

    $\frac{12}{42} = \left(\frac{24}{42}\right) \cdot \left(\frac{24}{42}\right)$

    $\frac{4}{14} \neq \left(\frac{4}{7}\right) \cdot \left(\frac{4}{7}\right)$

    not independent

11. Use the sample space from number 18 to determine whether randomly spinning blue and then green are independent events.
    Event A: Blue on first spin
    Event B: Green on second spin

    $P(\text{Blue and Green}) = P(\text{Blue}) \cdot P(\text{green})$

    $\frac{1}{16} = \left(\frac{4}{16}\right) \cdot \left(\frac{4}{16}\right)$

    $\frac{1}{4} \cdot \frac{1}{4}$

    $\frac{1}{16}$

    independent

/3

12. Use the sample space from number 19 to determine whether randomly selecting a green apple first and randomly selecting a green apple second are independent events.

P(green and green) = P(green) · P(green)

$\frac{6}{12} = \frac{9}{12} \cdot \frac{9}{12}$

$\frac{1}{2} = \frac{3}{4} \cdot \frac{3}{4}$ $\frac{1}{2} \neq \frac{9}{16}$ Not independent

Event A: green apple first

Event B: green apple second

13. Use the sample space from number 20 to determine whether randomly guessing question 1 correctly and question 2 correctly are independent events.

P(a and b) = P(a) · P(b)

$\frac{8}{32} = \frac{16}{32} \cdot \frac{16}{32}$

$\frac{1}{4} = \frac{1}{4}$ Independent

Event A: correct on question 1

Event B: correct on question 2

14. Use the sample space from number 21 to determine whether randomly selecting a white rose first and randomly selecting a white rose second are independent events.

P(white and white) = P(white 1st) · P(white 2nd)

$\frac{12}{20} = \frac{16}{20} \cdot \frac{16}{20}$

$\frac{3}{5} = \frac{4}{5} \cdot \frac{4}{5}$ $\frac{3}{5} \neq \frac{16}{25}$ Not independent

Event A: White Rose First

Event B: White Rose Second

Determine if the events are independent or dependent. Then use the appropriate formula to determine their probability.

Independent Events: $P(A \text{ and } B) = P(A) \cdot P(B)$

Dependent Events: $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

15. You play a game that involves drawing two numbers from a hat. There are 25 pieces of paper numbered from 1 to 25 in the hat. Each number is replaced after it is drawn. Find the probability that you will draw the 3 on your first draw and a number greater than 10 on your second draw.

P(3 and 10) = P(3) · P(# > 10)

$\frac{1}{25} \cdot \frac{15}{25}$

$\frac{1}{25} \cdot \frac{3}{5} = \frac{3}{125}$

Event A: drawing a 3

Event B: drawing a number > 10

Dep/Ind: Independent

Probability: 3/125

16. A drawer contains 12 white socks and 8 black socks. You randomly choose 1 sock and do not replace it. Then you randomly choose another sock. Find the probability that both socks are white.

P(A and B) = P(A) · P(B|A)

$\frac{12}{20} \cdot \frac{11}{19}$

$\frac{3}{5} \cdot \frac{11}{19} = \frac{33}{95}$

Event A: white sock first

Event B: white sock second

Dep/Ind: Dependent

Probability: 33/95

17. A word game has 100 tiles, 98 of which are letters and 2 of which are blank. The numbers of tiles of each letter are shown. You randomly draw 1 tile, set it aside, and then randomly draw another tile. Find the probability that the first tile is a consonant and the second tile is a vowel.

A - 9, B - 2, C - 2, D - 4, E - 12, F - 2, G - 3, H - 2, I - 9, J - 1, K - 1, L - 4, M - 2, N - 6, O - 8, P - 2, Q - 1, R - 6, S - 4, T - 6, U - 4, V - 2, W - 2, X - 1, Y - 2, Z - 1, Blank - 2

Vowels: 9 + 12 + 9 + 8 + 4 = 42

Consonants: 56

Blanks: 2

P(A and B) = P(A) · P(B|A)

$\frac{56}{100} \cdot \frac{42}{99}$

$\frac{14}{25} \cdot \frac{14}{33} = \frac{196}{825}$

Event A: Consonant

Event B: Vowel

Dep/Ind: Dependent

Probability: 196/825

18. Events A and B are independent. Suppose $P(B) = 0.4$ and $P(A \text{ and } B) = 0.13$. Find $P(A)$.

$P(A \text{ and } B) = P(A) \cdot P(B)$

$0.13 = P(A) \cdot (.4)$

$.325 = P(A)$

19. Events A and B are dependent. Suppose $P(B \mid A) = 0.6$ and $P(A \text{ and } B) = 0.15$. Find $P(A)$.

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

$0.15 = P(A) \cdot (0.6)$

$.25 = P(A)$

20. Events A and B are independent. Describe and correct the error in finding $P(A \text{ and } B)$.

✗ $P(A) = 0.6 \quad P(B) = 0.2$

$P(A \text{ and } B) = 0.6 + 0.2 = 0.8$

Probabilities should have been multiplied

$(.6)(.2) = .12$

21. A shelf contains 3 fashion magazines and 4 health magazines. You randomly choose one to read, set it aside, and randomly choose another for your friend to read. Describe and correct the error in finding the probability that the first magazine is fashion and the second magazine is health.

✗ $P(A) = \frac{3}{7} \quad P(B|A) = \frac{4}{7}$

$P(A \text{ and } B) = \frac{3}{7} \cdot \frac{4}{7} = \frac{12}{49} \approx 0.245$

Dependent events. Denominator should have decreased by 1

$\frac{3}{7} \cdot \frac{4}{6} = \frac{2}{7}$

22. You randomly select three cards from a standard deck of 52 playing cards. What is the probability that all three cards are face cards when...

a. you replace each card before selecting the next card.

$\frac{12}{52} \cdot \frac{12}{52} \cdot \frac{12}{52} = \left(\frac{3}{13}\right)^3 = \frac{27}{2197}$

b. you do not replace each card before selecting the next card.

$\frac{12}{52} \cdot \frac{11}{51} \cdot \frac{10}{50} = \frac{3}{13} \cdot \frac{11}{51} \cdot \frac{1}{5} = \frac{33}{3315}$

23. At a school, 43% of students attend the homecoming football game. Only 23% of students go to the game and the homecoming dance. What is the probability that a student who attends the football game also attends the dance?

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

$.23 = .43 \cdot P(B|A)$

$\approx .53 = P(B|A)$

Event A: attend football Game 43%

Event B: attend the dance

Ind/Dep: Dependent

Probability: .53

24. A meteorologist claims that there is a 70% chance of rain. When it rains, there is a 75% chance that your softball game will be rescheduled. Your friend believes the game is more likely to be rescheduled than played. Is your friend correct? Give a mathematical explanation using the formulas for independent or dependent events.

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

$P(\text{rain and reschedule}) = (.70) \cdot (.75)$

$P(\text{rain and reschedule}) = 52.5\%$

Event A: Rains .70

Event B: Game is rescheduled

Ind/Dep: Dependent

Probability: 52.5%

Yes 52.5% chance it is rescheduled

due tomorrow - questions on yesterday's ws??

Name:_____Hour_____ **Probability With and Without Replacement & Independent or Dependent**

Independent Events: $P(A \text{ and } B) = P(A) \cdot P(B)$

Dependent Events: $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

You have a jar of gumballs: 4 red, 9 green, 8 blue, 6 yellow, and 3 white. One gumball is drawn randomly. Find the following Probabilities and write as a reduced fraction and as a percent.

1. P(white)
2. P(green)
3. P(blue ∪ yellow)
4. P($\overline{Red}$)

You roll a 6 sided die one time. Find the following probabilities. Write as a reduced fraction and as a percent.

5. P(7)
6. P(1 ∪ 2)
7. P(odd number)
8. P($\bar{6}$)

In your math class there are the following white board markers at the board: 2 green, 2 blue, 2 red, 1 purple, and 1 black. One student randomly chooses a marker and then replaces it. The second student then chooses a marker. What is the probability of the students randomly choosing the colors listed below? Write as a percent

9. P(green, blue)
10. P(red, purple)
11. P(black, black)
12. P(purple, green)

In your sock drawer you have 10 pairs of white socks, 4 pairs of black, and 2 pairs of brown. You randomly choose a pair of socks each day. Sometimes you don't replace them because they are dirty. You choose another pair of socks the next day. Find the probability of the following situations. Write the probability as a reduced fraction.

13. P(white, white) with replacement
14. P(white, black) without replacement
15. P(black, brown) without replacement
16. P(brown, brown) without replacement
17. P(white, white, white) with replacement
18. P(white, black, brown) with replacement
19. P(black, white, white) without replacement
20. P(brown, brown, brown) with replacement

Determine if the following events are dependent or independent. Then calculate the probability of each.

21. Selecting a glazed donut from an assortment of twelve donuts, 4 glazed, 4 with sprinkles, 4 maple bars, and 4 cake donuts. Then eating it, and then selecting a maple bar from the same box.

22. Given a bag of marbles with 2 red, 3 green and 2 blue. What is the probability of choosing a red marble keeping it, then choosing another red marble?

23. Rolling a 3 on a dice and then drawing a red card from a deck of cards.

24. You are choosing two cards from a deck. The first card is a queen, if you keep that card what is the probability that the second card is a face card?

25. Flipping a coin, and getting tails both times.

Independent Events: $P(A \text{ and } B) = P(A) \cdot P(B)$

Dependent Events: $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

26. Events A and B are independent. Find the missing probability

P(A) = .73	P(A) = .15	P(A) =
P(B) =	P(B) = .62	P(B) =.85
P(A and B) = .28	P(A and B) =	P(A and B) = .33

27. Events A and B are dependent. Find the missing probability

P(A) = .37	P(A) =	P(A) = .41
P(B|A) = .18	P(B|A) = .61	P(B|A) =
P(A and B) =	P(A and B) = .57	P(A and B) = .35

28. In a random sample of 1000 Ridgeline students, a study found that 40.7% were involved in a club and 21.8% studied 4 hours per day. When they reported their findings, the research group indicated that 41.4% of students were either involved in a club or they studied 4 hours per day. Given this information, what is the probability that a student is involved in a club and studies 4 hours per day?

29. In a random sample of 150 Ridgeline Cross Country runners, a study found that 72% of them ran five days a week and 18% of them did their core exorcises 5 days a week. When they reported their findings, the research group indicated that 79.4% of the athletes either ran or did core exorcises five days a week. Given this information, what is the probability that an athlete ran and did their core exorcises five days a week?

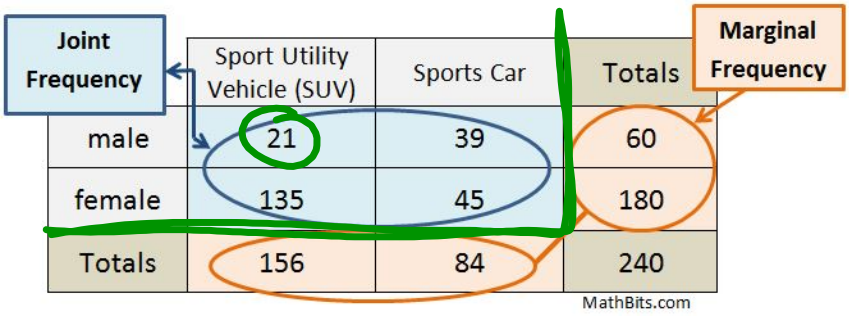
Essential Question

How can you construct and interpret a two-way table?

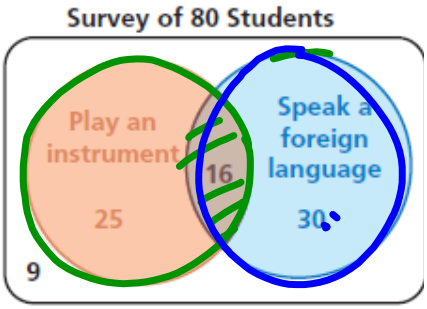
A Two - Way Table displays data collected from the same source that belongs to two different categories...

The entries in the table are called "Joint Frequencies"

Row and column totals are called "Marginal Frequencies"



The Venn diagram shows the results of a survey in which 80 students were asked whether they play a musical instrument and whether they speak a foreign language. Use the Venn diagram to complete the two-way table.



	Play an Instrument	Do Not Play an Instrument	Total
Speak a Foreign Language	16	30	46
Do Not Speak a Foreign Language	25	9	34
Total	41	39	80

joint freq.
marg. freq

Use the previous table to answer the questions

	Play an Instrument	Do Not Play an Instrument	Total
Speak a Foreign Language	16	30	46
Do Not Speak a Foreign Language	25	9	34
Total	41	39	80

a. How many students play an instrument? 41

b. How many students play an instrument and speak a foreign language? 16

c. How many students do not play an instrument and do not speak a foreign language? 9

One student is selected at random from the 80 students who took the survey.
Find the probability that the student:

a. plays an instrument. $41/80 \approx 51\%$

b. plays an instrument and speaks a foreign language. $16/80 = 20\%$

c. does not play an instrument and does not speak a foreign language. $9/80 = 11\%$

Core Concept

Relative and Conditional Relative Frequencies

A **joint relative frequency** is the ratio of a frequency that is not in the total row or the total column to the total number of values or observations.

A **marginal relative frequency** is the sum of the joint relative frequencies in a row or a column.

A **conditional relative frequency** is the ratio of a joint relative frequency to the marginal relative frequency. You can find a conditional relative frequency using a row total or a column total of a two-way table.

Use the survey results to make a two-way table that shows the joint and marginal ***relative*** frequencies.

	Play an Instrument	Do Not Play an Instrument	Total
Speak a Foreign Language	16	30	46
Do Not Speak a Foreign Language	25	9	34
Total	41	39	80

Joint freq: 16 30 / 25 9

	Play an Instrument	Do Not Play an Instrument	Total
Speak a Foreign Language	$\frac{16}{80} = .2$	$\frac{30}{80} = .375$	$\frac{46}{80} = .575$
Do Not Speak a Foreign Language	$\frac{25}{80} = .3125$	$\frac{9}{80} = .1125$	$\frac{34}{80} = .425$
Total	$\frac{41}{80} = .5125$	$\frac{39}{80} = .4875$	$\frac{80}{80} = 1$

Conditional Probability... P(A|B)

P(A given B) ← total

	Play an Instrument	Do Not Play an Instrument	Total
Speak a Foreign Language	16	30	46
Do Not Speak a Foreign Language	25	9	34
Total	41	39	80

P(Speak | Play) = $\frac{16}{41}$

P(Play | Don't Speak) = $\frac{25}{34}$

P(Don't Speak | Play) = $\frac{25}{41}$

P(Don't Play | Speak) = $\frac{30}{46}$

P(Speak | Don't Play) =

P(Play | Speak) =

Use the survey results to make a two-way table that shows the conditional relative frequencies based on the ***row totals***. Interpret them!

	Play an Instrument	Do Not Play an Instrument	Total
Speak a Foreign Language	16	30	46
Do Not Speak a Foreign Language	25	9	34
Total	41	39	80

	Play an Instrument	Do Not Play an Instrument	Total
Speak a Foreign Language	.2	.375	.575
Do Not Speak a Foreign Language	.3125	.1125	.425
Total	.5125	.4875	1

Use either table...

	Play an Instrument	Do Not Play an Instrument
Speak a Foreign Language	$\frac{.2}{.575} = .35$	$\frac{.375}{.575} = .65$
Do Not Speak a Foreign Language	$\frac{.3125}{.425}$	$\frac{.1125}{.425}$

row total → given

$P(\text{play} \mid \text{speak})$

$P(\overline{\text{play}} \mid \text{speak})$

In another survey similar to the one above, 106 juniors and 114 seniors respond. Of those, 42 juniors and 77 seniors plan on attending. Organize these results in a two-way table. Then find and interpret the marginal frequencies.

		Attendance		
		Attending	Not Attending	Total
Class	Junior			
	Senior			
	Total			

Use the survey results to make a two-way table that shows the joint and marginal ***relative*** frequencies.

		Attendance		
		Attending	Not Attending	Total
Class	Junior	42	64	106
	Senior	77	37	114
	Total	119	101	220

		Attendance		
		Attending	Not Attending	Total
Class	Junior			
	Senior			
	Total			

Use the survey results to make a two-way table that shows the *conditional relative frequencies* based on the **attendance totals**. Interpret them!

		Attendance		
		Attending	Not Attending	Total
Class	Junior	42	64	106
	Senior	77	37	114
	Total	119	101	220

		Attendance	
		Attending	Not Attending
Class	Junior		
	Senior		

A satellite TV provider surveys customers in three cities. The survey asks whether they would recommend the TV provider to a friend. The results, given as joint relative frequencies, are shown in the two-way table.

		Location			
		Glendale	Santa Monica	Long Beach	Total
Response	Yes	0.29	0.27	0.32	.88
	No	0.05	0.03	0.04	.12
Total		.34	.30	.36	1

a. What is the probability that a randomly selected customer who is located in Glendale will recommend the provider?

P(Yes | Glendale)

$\frac{.29}{.34} = .85$

at is the probability that a randomly
ed customer who will not recommend
ovider is located in Long Beach?

P(LB | No)

$\frac{.04}{.12} = .33$

c. What is the probability that a randomly selected customer who is located in Santa Monica will not recommend the provider to a friend

P(not | SM) =

$\frac{.03}{.30} = .1 = 10\%$

Determine whether recommending the provider to a friend and living in Long Beach are *independent* events.

Event A Yes-recommend

Event B Long Beach

		Location			
		Glendale	Santa Monica	Long Beach	Total
Response	Yes	0.29	0.27	0.32	0.88
	No	0.05	0.03	0.04	0.12
	Total	0.34	0.3	0.36	1

$P(A \text{ and } B) = P(A) \cdot P(B)$

$.32 = (.88)(.36)$

$.32 \neq .3168$

Not Indep

Determine whether recommending the provider to a friend and living in Santa Monica are *independent* events.

Event A Yes-rec.

Event B Santa Monica

		Location			
		Glendale	Santa Monica	Long Beach	Total
Response	Yes	0.29	0.27	0.32	0.88
	No	0.05	0.03	0.04	0.12
Total		0.34	0.3	0.36	1

$P(A \text{ and } B) = P(A) \cdot P(B)$

$.27 = (.88) \cdot (.3)$

$.27 \neq .264$ Not Independent

11. **MODELING WITH MATHEMATICS** A survey finds that 110 people ate breakfast and 30 people skipped breakfast. Of those who ate breakfast, 10 people felt tired. Of those who skipped breakfast, 10 people felt tired. Make a two-way table that shows the conditional relative frequencies based on the breakfast totals. *(See Example 3.)*

due Friday
5.3 online hw Pg 300-302 #s 1-11, 13-16, 20