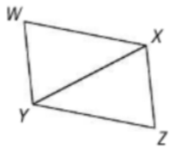
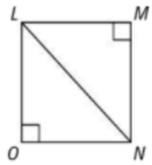
Bell Ringer

Section 10.4 – CPCTC

1. **Given:** $\overline{WX} \cong \overline{ZY}$, $\overline{WY} \cong \overline{ZX}$
   Prove: $\angle W \cong \angle Z$



2. **Given:** $\angle ONL \cong \angle MLN$, $\angle O$ and $\angle M$ are right angles.
   Prove: $\overline{LM} \cong \overline{NO}$



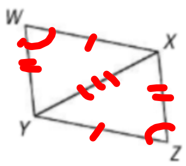
3. Solve for x. $2^x = 1{,}024$

4. Solve for x. $\frac{1}{81} = 3^x$

Solutions

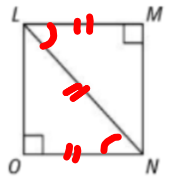
Section 10.4 – CPCTC

1. **Given:** $\overline{WX} \cong \overline{ZY}$, $\overline{WY} \cong \overline{ZX}$
Prove: $\angle W \cong \angle Z$



$\overline{WX} \cong \overline{ZY}$ – Given
$\overline{WY} \cong \overline{ZX}$ – Given
$\overline{YX} \cong \overline{YX}$ – Reflexive Property
$\Delta WXY \cong \Delta ZYX$ – SSS
$\angle W \cong \angle Z$ – CPCTC

2. **Given:** $\angle ONL \cong \angle MLN$, $\angle O$ and $\angle M$ are right angles.
Prove: $\overline{LM} \cong \overline{NO}$



$\angle ONL \cong \angle MLN$ – Given
$\angle O \cong \angle M$ – All right angles are congruent
$\overline{LN} \cong \overline{LN}$ – Reflexive Property
$\Delta LMN \cong \Delta NOL$ – AAS
$\overline{LM} \cong \overline{NO}$ – CPCTC

3. Solve for x. $2^x = 1{,}024$

x = 10

4. Solve for x. $\frac{1}{81} = 3^x$

x = -4

$3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

$\frac{1}{81} = 81^{-1}$

correct 10.2, 10.3 and 10.4

10.2 #s 2-6, 8-11, 13-23

1. **a.** Given
b. Refl. Prop. of ≅
c. $\triangle JKM$
d. $\triangle LMK$

2. F is the midpt. of $\overline{GI}$ (Given), so $\overline{IF} \cong \overline{GF}$ because a midpt. divides a segment into two ≅ segments. The other two pairs of sides are given as ≅, so $\triangle EFI \cong \triangle HFG$ by SSS.

3. You need to know $\overline{LG} \cong \overline{MN}$; the diagram shows that $\overline{LT} \cong \overline{MQ}$ and $\angle L \cong \angle M$. $\angle L$ is included between $\overline{LG}$ and $\overline{LT}$, and $\angle M$ is included between $\overline{MN}$ and $\overline{MQ}$.

4. You need to know $\overline{RS} \cong \overline{WU}$; the diagram shows that $\angle R \cong \angle W$ and $\overline{RT} \cong \overline{WV}$. $\angle R$ is included between $\overline{RT}$ and $\overline{RS}$, and $\angle W$ is included between $\overline{WV}$ and $\overline{WU}$. Alternatively, you need to know that $\angle T \cong \angle V$.

5. Not enough information; the congruent vertical angles TQP and RQS are not included by the pairs of ≅ sides.

6. SAS; the ≅ angles BAC and DCA are included by the pairs of sides $\overline{AB} \cong \overline{CD}$ (Given) and $\overline{AC} \cong \overline{AC}$ (Refl. Prop. of ≅).

7. **a.** $\angle PEN$ (or $\angle E$)
b. $\angle NPE$ (or $\angle P$)

8. **a.** $\overline{HA}$ and $\overline{HT}$
b. $\overline{TH}$ and $\overline{TA}$

9. SAS

10. SSS

11. Answers may vary. Sample: Alike: Both use three pairs of ≅ parts to prove △s ≅. Different: SSS uses three pairs of ≅ sides, while SAS uses two pairs of ≅ sides and their ≅ included ∠s.

12. No; the ≅ ∠s are not included between the pairs of ≅ sides.

13. No; the △s have the same perimeter, but the three side lengths of one △ are not necessarily = to the three side lengths of the other △, so you cannot use SSS. There is no information about the ∠s of the △s, so you cannot use SAS.

14. If the 40° ∠ is *always* included between the two 5-in. sides, then all the triangles will be congruent by SAS. In other cases, the triangles may or may not be congruent.

23. $\triangle ANG \cong \triangle RWT$ by SAS.

24. Not enough information; you need $\overline{DY} \cong \overline{TK}$ to show the △s are ≅ by SSS, or you need $\angle H \cong \angle P$ to show the △s are ≅ by SAS.

25. $\triangle JEF \cong \triangle SFV$ (or $\triangle JEF \cong \triangle SVF$) by SSS

~~**26.** Not necessarily; the ≅ ∠ are not included between the pairs of ≅ sides.~~

27. $\overline{GK}$ bisects $\angle JGM$ (Given), so $\angle JGK \cong \angle MGK$ (Def. of ∠ bisector). $\overline{GJ} \cong \overline{GM}$ (Given) and $\overline{GK} \cong \overline{GK}$ (Refl. Prop. of ≅), so $\triangle GJK \cong \triangle GM$ by SAS.

10.3 #s 1-3, 5 ,7-15, 20

1. $\triangle PRQ \cong \triangle VWX$

2. $\triangle ABC \cong \triangle EDF$

3. **a.** Reflexive

b. ASA

5. **a.** Vert. $\angle$s are $\cong$.

b. Given

c. $\overline{TQ} \cong \overline{RQ}$

d. AAS

7. $\triangle PMO \cong \triangle NMO$ by ASA.

8 $\triangle UST \cong \triangle RTS$ by AAS.

9. $\overline{RS}$

10. $\angle N$, $\angle O$

11. ASA

12. AAS

13. Answers may vary. Sample: Alike: Both postulates use three pairs of $\cong$ corresp. parts. Different: To use the ASA Postulate, the sides must be included between the pairs of corresp. angles, while to use the SAS Postulate, the angles must be included between the pairs of corresp. sides.

14. $\overline{LM}$ is not included between the pairs of $\cong$ corresp. angles.

15. $\angle F \cong \angle G$, $\angle D \cong \angle H$

20. No; the common side is included between the two $\cong$ $\angle$s in one $\triangle$, but it is not included between the $\cong$ $\angle$s in the other $\triangle$.

10.4 #s 1, 3-5, 7, 10-12

1. $\triangle KLJ \cong \triangle OMN$ by SAS; $\overline{KJ} \cong \overline{ON}$, $\angle K \cong \angle O$, $\angle J \cong \angle N$.

3. **a.** $\triangle KRA$

b. ASA

c. Corresp. parts of $\cong$ △s are $\cong$.

4. SAS; so $\overline{EA} \cong \overline{MA}$ because corresp. parts of $\cong$ △s are $\cong$.

5. SSS; so $\angle U \cong \angle E$ because corresp. parts of $\cong$ △s are $\cong$.

7. $\triangle KHL \cong \triangle NHM$ by AAS Thm.

10–12. Check students' diagrams.

10. $\overline{KL}$ bisects $\angle PKQ$, so $\angle PKL \cong \angle QKL$. $\overline{KL} \cong \overline{KL}$ by Refl. Prop. of $\cong$. $\triangle PKL \cong \triangle QKL$ by SAS, so $\angle P \cong \angle Q$ because corresp. parts of $\cong$ △s are $\cong$.

11. From the def. of $\perp$ bisector, $\overline{PL} \cong \overline{QL}$ and $\angle PLK \cong \angle QLK$ because all rt. ∠s are $\cong$. Since $\overline{KL} \cong \overline{KL}$, by Refl. Prop. of $\cong$ then $\triangle PKL \cong \triangle QKL$ by SAS, and $\angle P \cong \angle Q$ because corresp. parts of $\cong$ △s are $\cong$.

12. $\angle PLK \cong \angle QLK$ because $\perp$ lines form rt. ∠s, and all rt. ∠s are $\cong$. From the $\angle$ bisector, $\angle PKL \cong \angle QKL$. So with $\overline{KL} \cong \overline{KL}$ by the Refl. Prop. of $\cong$, $\triangle PKL \cong \triangle QKL$ by ASA and $\angle P \cong \angle Q$ because corresp. parts of $\cong$ △s are $\cong$.

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10-5 Isosceles and Equilateral Triangles

G.CO.10 Prove theorems about triangles . . . base angles of isosceles triangles are congruent . . . Also **G.CO.13, G.SRT.5**

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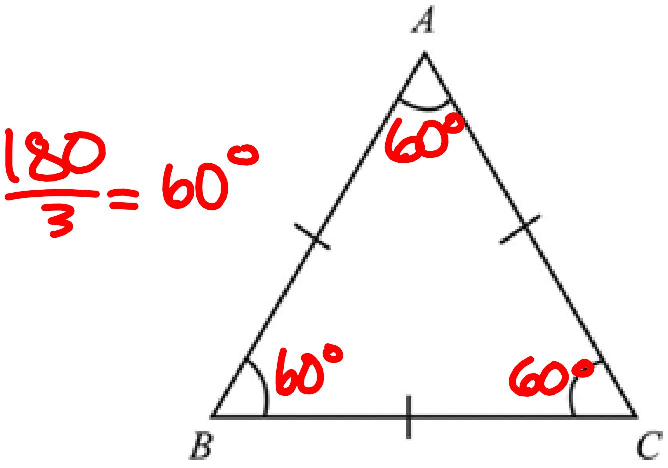
Equilateral Triangle



Solve for x. Find all side lengths and angle measures.

x = 4

AB = 11

BC = 11

AC = 11

<A = 60°

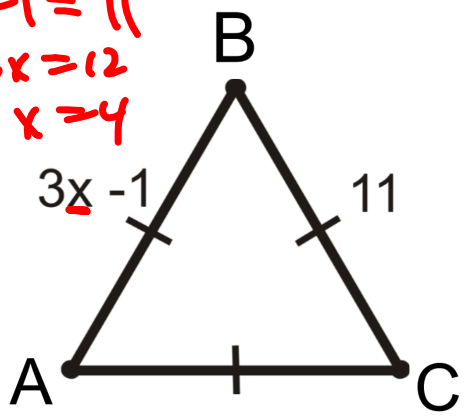
<B = 60°

<C = 60°

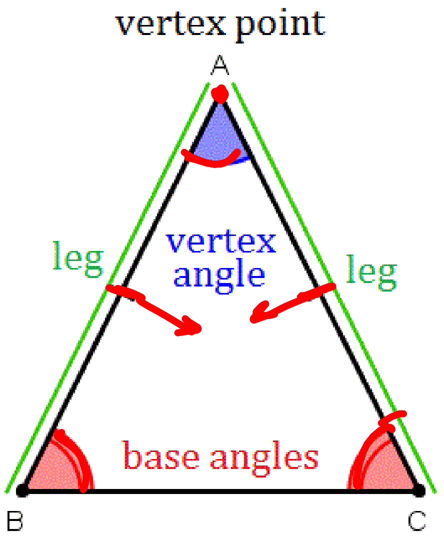
$3x - 1 = 11$

$3x = 12$

$x = 4$



Isosceles Triangle



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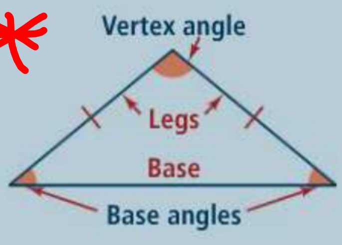
In the Solve It, you classified a triangle based on the lengths of its sides. You can also identify certain triangles based on information about their angles. In this lesson, you will learn how to use and apply properties of isosceles and equilateral triangles.

Essential Understanding The angles and sides of isosceles and equilateral triangles have special relationships.

Isosceles triangles are common in the real world. You can * frequently see them in structures such as bridges and buildings, as well as in art and design. The congruent sides of an isosceles triangle are its **legs.** The third side is the **base.** The two congruent legs form the **vertex angle.** The other two angles are the **base angles.**



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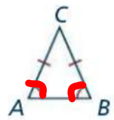
pg 558

take note

Theorem 3 Isosceles Triangle Theorem

Theorem	If . . .	Then . . .
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.	$\overline{AC} \cong \overline{BC}$	$\angle A \cong \angle B$



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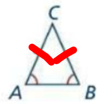
take note

Theorem 4 Converse of the Isosceles Triangle Theorem

Theorem	If . . .	Then . . .
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.	$\angle A \cong \angle B$	$\overline{AC} \cong \overline{BC}$



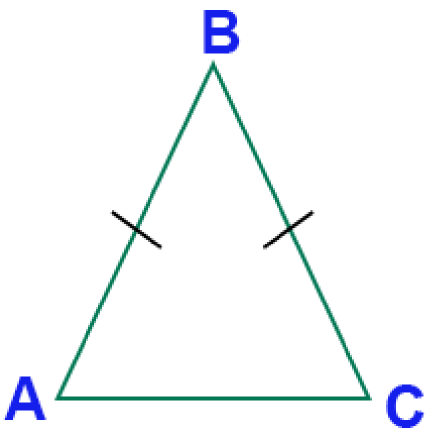
You will prove Theorem 4 in Exercise 22.

What can we conclude about any of the angles??

How do you know?

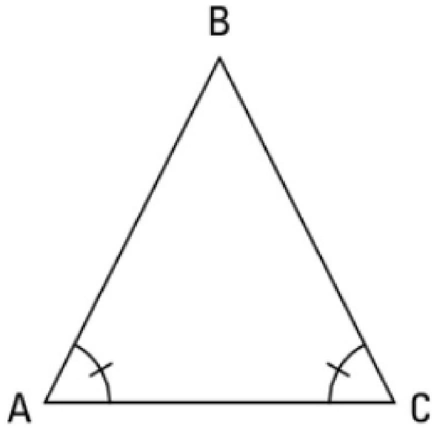
$\angle A \cong \angle C$

by I.T.T.



What can you conlcude about any of the sides?

How do you know?



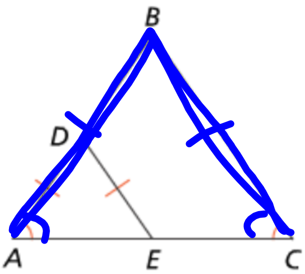
$\overline{BA} \cong \overline{BC}$

by converse of I.T.T

not in book

Problem 1 Using the Isosceles Triangle Theorems

A Is $\overline{AB}$ congruent to $\overline{CB}$? Explain.

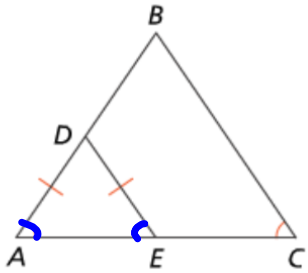


Yes, by converse of I.T.T

not in book

Problem 1 Using the Isosceles Triangle Theorems

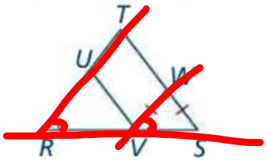
B Is $\angle A$ congruent to $\angle DEA$? Explain.



Yes, by I.T.T

Got it pg 559

Got It? **a.** Is $\angle WVS$ congruent to $\angle S$? Is $\overline{TR}$ congruent to $\overline{TS}$? Explain.



Yes, by I.T.T.

b. Reasoning Can you conclude that $\triangle RUV$ is isosceles? Explain.

Not enough info...

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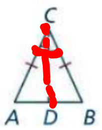
An isosceles triangle has a certain type of symmetry about a line through its vertex angle.

take note

Theorem 5

Theorem	If . . .	Then . . .
If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.	$\overline{AC} \cong \overline{BC}$ and $\angle ACD \cong \angle BCD$	$\overline{CD} \perp \overline{AB}$ and $\overline{AD} \cong \overline{BD}$



You will prove Theorem 5 in Exercise 25.

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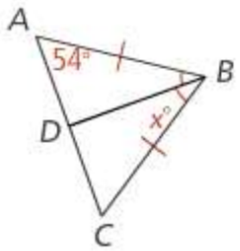
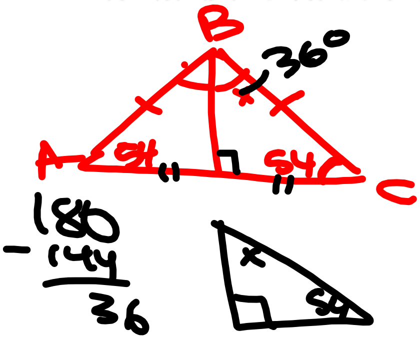
not in book

Problem 2 Using Algebra

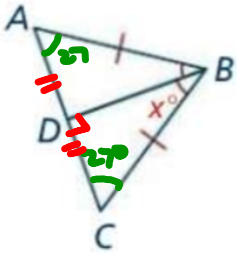
What is the value of x?

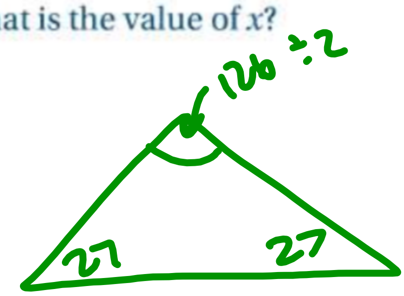




Got it pg 561

Got It? Suppose $m\angle A = 27$. What is the value of x?





$180 - 117 = 63°$

$27 + 27 = 54$

$180 - 54 = 126 \div 2 = 63$

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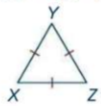
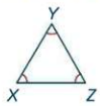
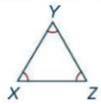
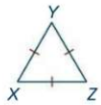
A **corollary** is a theorem that can be proved easily using another theorem. Since a corollary is a theorem, you can use it as a reason in a proof.

take note

Corollary to Theorem 3

Corollary	If . . .	Then . . .
If a triangle is equilateral, then the triangle is equiangular.	$\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$ 	$\angle X \cong \angle Y \cong \angle Z$ 

Corollary to Theorem 4

Corollary	If . . .	Then . . .
If a triangle is equiangular, then the triangle is equilateral.	$\angle X \cong \angle Y \cong \angle Z$ 	$\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$ 

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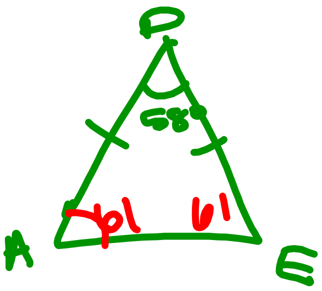
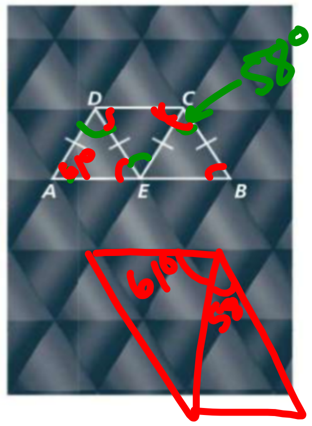
Got it pg 562

Got It? Suppose the triangles in Problem 3 are isosceles triangles, where $\angle ADE$, $\angle DEC$, and $\angle ECB$ are vertex angles. If the vertex angles each have a measure of 58, what are $m\angle A$ and $m\angle BCD$?

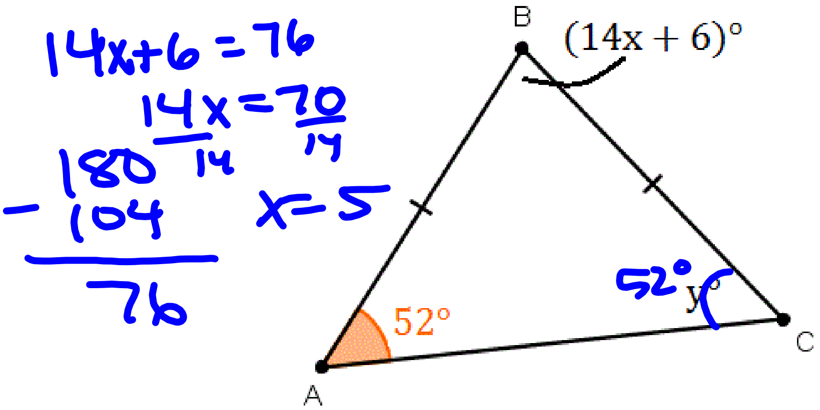


$$180 - 58 = 122 \div 2$$

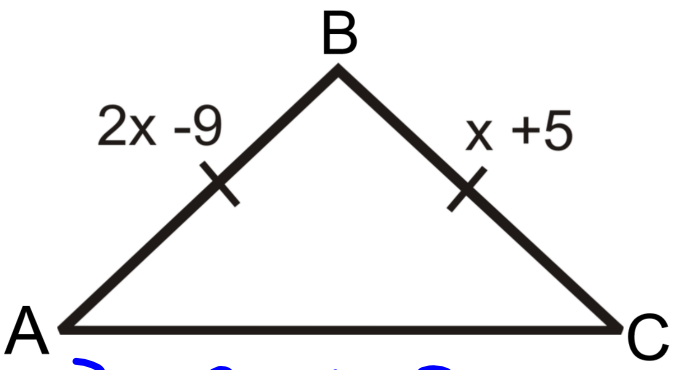
$119°$



Find the values of x and y



Find the lengths of AB and BC



$2x - 9 = x + 5$

hw 10.5 #s 1-18